

Extrapolation For Bin16

In support of comment #R2-191

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Current Extrapolation for bin 16

Based on linear model of $\log_{10}$ of $H_m(k)$, $H_m(k)$ = portion of bin k from total #FEC blocks

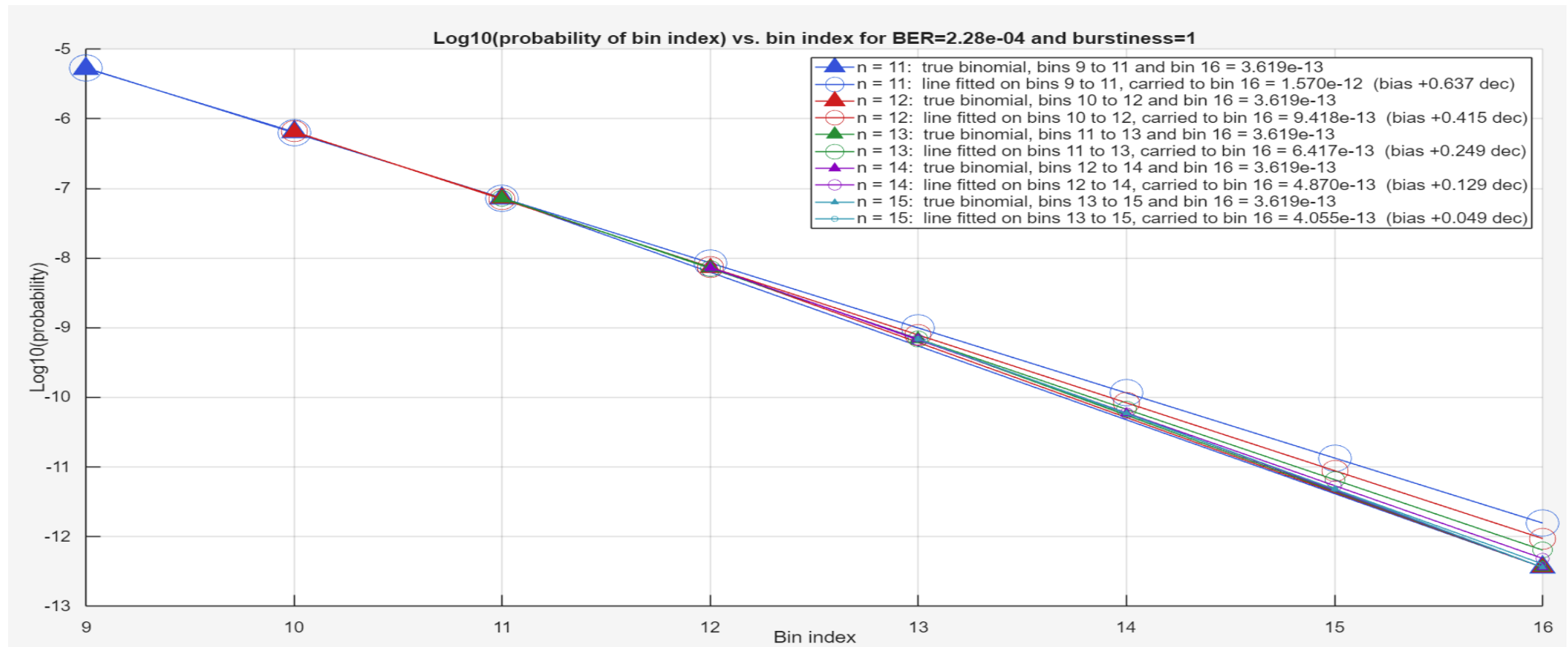
- Quotation from 180.9.14 Receiver sensitivity, page 492 :

"NOTE—In order to predict whether a receiver meets the BLER requirement in a short test time, extrapolation of the measured histogram (see 174A.9.3) to $H_m(16)$ can be performed. One way of doing this is using a linear fit of $\log_{10}(H_m(k))$ for $k = (n - 2)$ to n , where these values of k are the three highest bins having a count of 10 or more. If the slope of the histogram over the measured bins is increasing, it might be an indication of not meeting the BLER requirement. To verify that the slope is not increasing, the values of $(\log_{10}(H_m(k)) - \log_{10}(H_m(4))) / (k - 4)$ should be less than $(\log_{10}(H_m(4)) - \log_{10}(H_m(2))) / 2$ for each k greater than 4."

- However, the linear model of $\log_{10}$ of portion of uncorrelated errors is:
 - Biased towards false failures
 - With excessive scatter over random scenarios

The model is biased up (1)

- The extent of bias is meaningful towards false failures of marginally passing links
- The table below shows the model bias vs. binomial distribution values, **for bin16 and not BLER**



The model is biased up (2)

Summary table

- The extent of bias is meaningful towards false failures of marginally passing links
- The table below shows the model bias vs. binomial distribution values, **for bin16 and not BLER**

Max bin >=10	Min duration with BER = 2.28e-4	Extrapolated Bin 16	Model bias: $\text{Log}_{10} \left(\frac{\text{Extrapolated bin 16}}{\text{Target bin 16}} \right) = \text{Log}_{10} \left(\frac{\text{Extrapolated bin 16}}{3.619e-13^*} \right)$
11	>=4[Sec], Hm(11)=11	1.57e-12	0.637
12	>=36[Sec], Hm(12)=10	9.42e-13	0.415
13	>=6:17[Min:Sec], Hm(13)=10	6.42e-13	0.245
14	>=1:13[Hours:Min], Hm(14)=10	4.87e-13	0.129
15	>=14:57[Hours:Min], Hm(15)=10	4.05e-13	0.049

*Target bin16 with uncorrelated BER 2.28e-4 = 3.619e-13

The Monte Carlo Analysis (1)

- Draw 10,000 cases of Histogram bins $H_m(k)$, per 3 durations X 3 BER
 - Durations: 60[Sec], 10[Min] and 60[Min], BER values: $2.28e-4$ (target), $2.0e-4$ and $2.177e-4$ (BLER that is $\frac{1}{2}$ vs. BLER with $2.28e-4$)
- Per case execute the 180.9.14 log10-linear extrapolation and the proposed binomial extrapolation

Bin 16 is exactly 16 errored symbols, and the fit uses three bins

180.9.14 fits the three highest bins holding 10 counts or more, then evaluates that line at $k = 16$

One run of $T = 3600$ s, $N = 1.41e11$ FEC blocks at uncorrelated BER $2.281e-4$: the 17 counters of 174A.9.3, blocks expected in each



1 Which bins qualify

A bin qualifies at 10 counts.
Bins 0 to 13 do here; bin 14 holds 8 and does not.
 $n = 13$, the highest qualifying bin of 1 to 15.

2 Fit three bins, $n-2$ to n

Least squares straight line of $\log_{10} H_m(k)$ over bins 11, 12 and 13 only.
Slope here -1.01 decades per bin.
Bin 16 takes no part in the fit.

3 Evaluate at $k = 16$

$H_m(16) =$ the line at $k = 16$, here $6.33e-13$.
It is a probability of exactly 16 errored symbols, being a point of the same pdf the fit was made on.

Exactly 16, not 16 or more

The counter of 174A.9.2 in bin 16 holds '16 or more'. That tail is the BLER, $3.84e-13$ here.
The extrapolation returns exactly 16 instead, $3.57e-13$, a factor 1.076 below that tail, 0.03 decades.
So the measured bin 16 and the extrapolated bin 16 are not the same quantity, though both are called bin 16.

Three bins, and only three

Not every bin above the noise: the three highest that qualify, $n-2$ to n .
A run reaching $n = 13$ carries the line across 3 bins to reach 16; a shorter test reaches $n = 11$ or 12 and carries it 4 or 5.
 n is a random variable, the bin holding 10 counts on one run holding 9 on the next, so which three bins are used changes run to run.

Numbers are the mean histogram of the cell, from $\text{binopdf}(k, 544, 2.27892e-3)$. One drawn run differs; that spread is what the Monte Carlo measures.

The Monte Carlo Analysis (2)

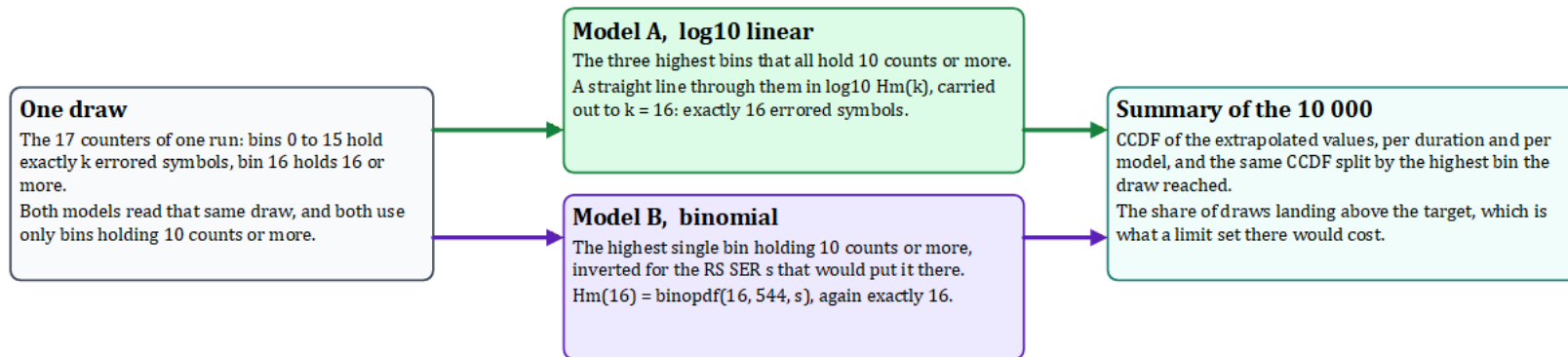
- Draw 10,000 cases of Histogram bins $H_m(k)$, per 3 durations X 3 BER
 - Durations: 60[Sec], 10[Min] and 60[Min], BER values: 2.281e-4 (target), 2.0e-4 and 2.177e-4 (BLER that is ½ vs. BLER with 2.28e-4)
- Per case execute the 180.9.14 log10-linear extrapolation and the proposed binomial extrapolation

3 BER points x 3 durations x 10 000 draws

every draw extrapolated to exactly 16 errored symbols, by two models, then summarized against the target

	T = 60 s, N = 2.34e9 blocks	T = 600 s, N = 2.34e10 blocks	T = 3600 s, N = 1.41e11 blocks
BER 2.281e-4 RSSER 2.279e-3, BLER 3.84e-13 the operating point, and the target	10 000 draws	10 000 draws	10 000 draws
BER 2.1772e-4 RSSER 2.175e-3, BLER 1.92e-13 0.500 x the BLER above, so halved	10 000 draws	10 000 draws	10 000 draws
BER 2.000e-4 RSSER 1.998e-3, BLER 5.40e-14 0.141 x the BLER above	10 000 draws	10 000 draws	10 000 draws

9 cells, 10 000 draws in each: 90 000 simulated tests, every one of them a full 17 bin histogram of one run



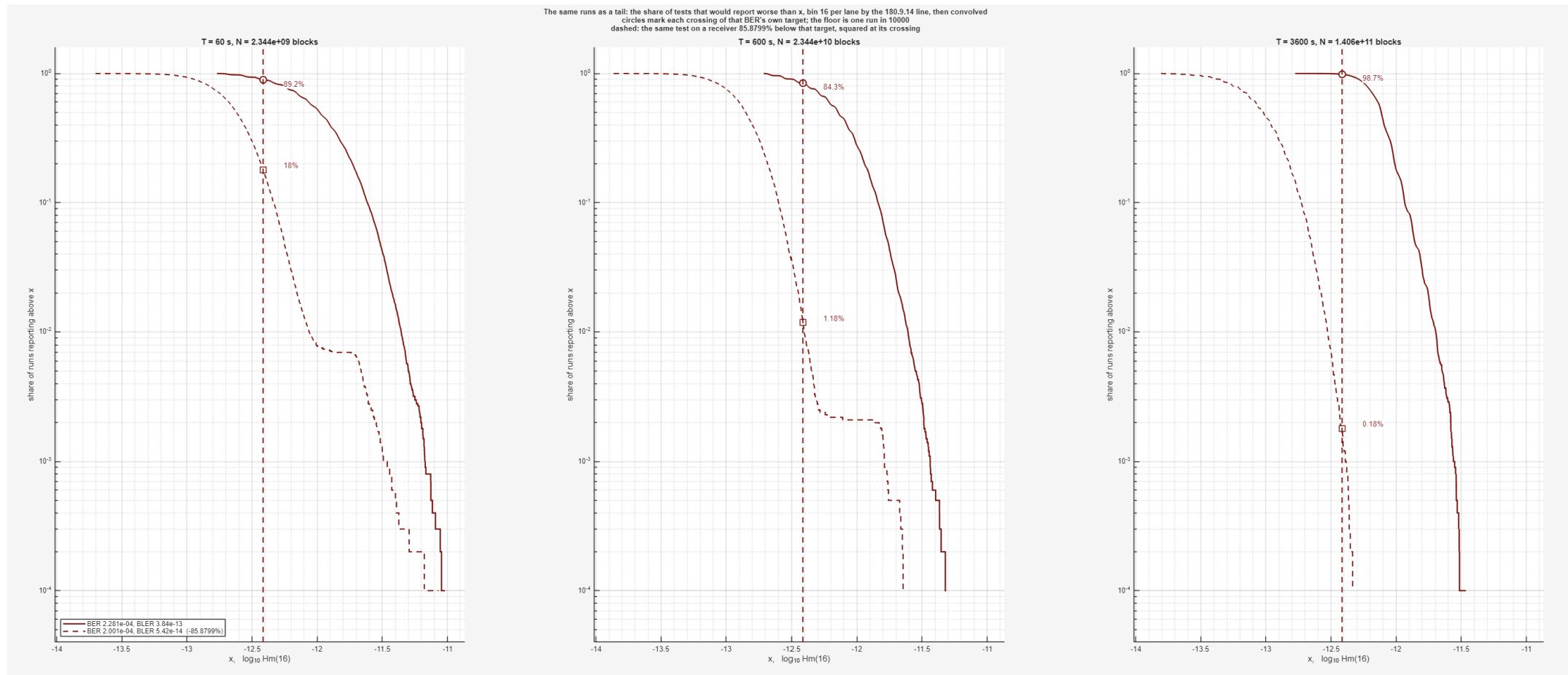
Bin 16 here is exactly 16 errored symbols, not 16 or more

Both models return the probability of exactly 16, which is 3.57e-13 at 2.281e-4, where the counter of 174A.9.2 holds the tail from 16 upward, 3.84e-13. The two are a factor 1.076 apart, 0.03 decades.

180.9.14 log10 linear model distribution (1)

Target BER=2.28e-4, leading to BLER=3.84e-13, vs. BER=2.0e-4 leading to BLER=5.42e-14 (-85.9%)

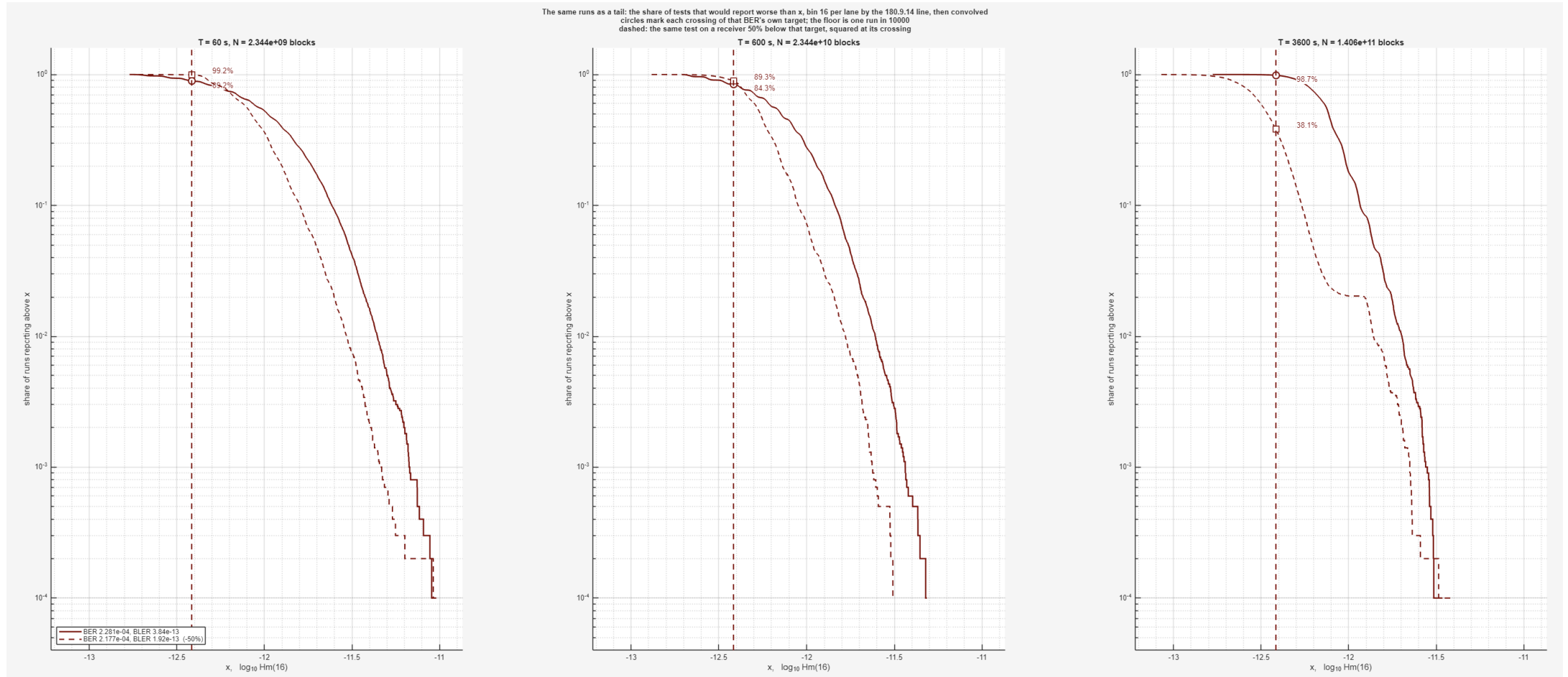
- Wide scatter over random scenarios, based on Monte Carlo draws
- Probability of more than 80% for failure of marginal links (uncorrelated BER of 2.281e-4), that should pass with 50%
 - For duration of 60[Sec], there's ~18% for false failure for links with uncorrelated BER of 2e-4, that should always pass
- The figures below show the CCDF (Complementary CDF = probability of X value and above/worse) of current extrapolation scheme for 1X links



180.9.14 log10 linear model distribution (2)

Target BER=2.281e-4, leading to BLER=3.84e-13, vs. BER=2.177e-4 leading to BLER=1.92e-13 (-50%)

- High probability of false failure for links with BLER that is 50% lower than target (1.92e-13), that should always pass
- The figures below show the CCDF (Complementary CDF = probability of X value and above/worse) of current extrapolation scheme for 1X links



Proposed Remedy – Model

Use directly the binomial distribution function which has no bias

- Equivalent RSSER(k): The RSSER for which the portion of histogram k fits the binomial distribution function

$$H_{\max}(k) = H(k)$$

$$\frac{n!}{k!(n-k)!} \text{RSSER}^k (1 - \text{RSSER})^{n-k} = \frac{\text{Number of FEC blocks with } k \text{ RS symbol errors}}{\text{Total Number of FEC blocks}}$$

- Proposed extrapolation:

- Find Equivalent RSSER for max nonzero histogram with count of 10 or more

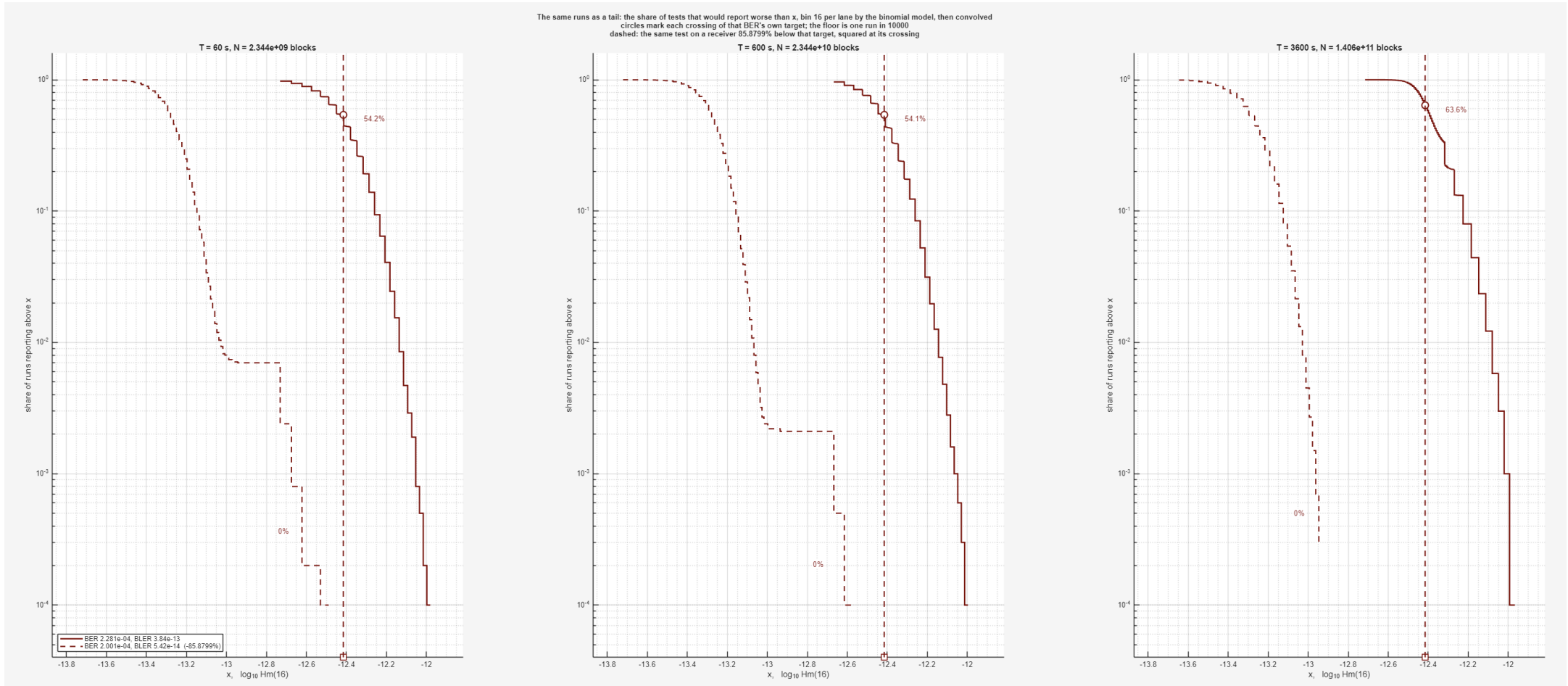
- Extrapolated bin 16 = $\sum_{k=16}^{544} \frac{544!}{k!(544-k)!} \text{Equivalent RSSER}^k (1 - \text{Equivalent RSSER})^{544-k}$

- Alternatively see if the following is satisfied for max nonzero histogram k with count of 10 or more

$$\frac{n!}{k!(n-k)!} \text{Target RSSER}^k (1 - \text{Target RSSER})^{n-k} = \frac{\text{Number of FEC blocks with } k \text{ RS symbol errors}}{\text{Total Number of FEC blocks}}$$

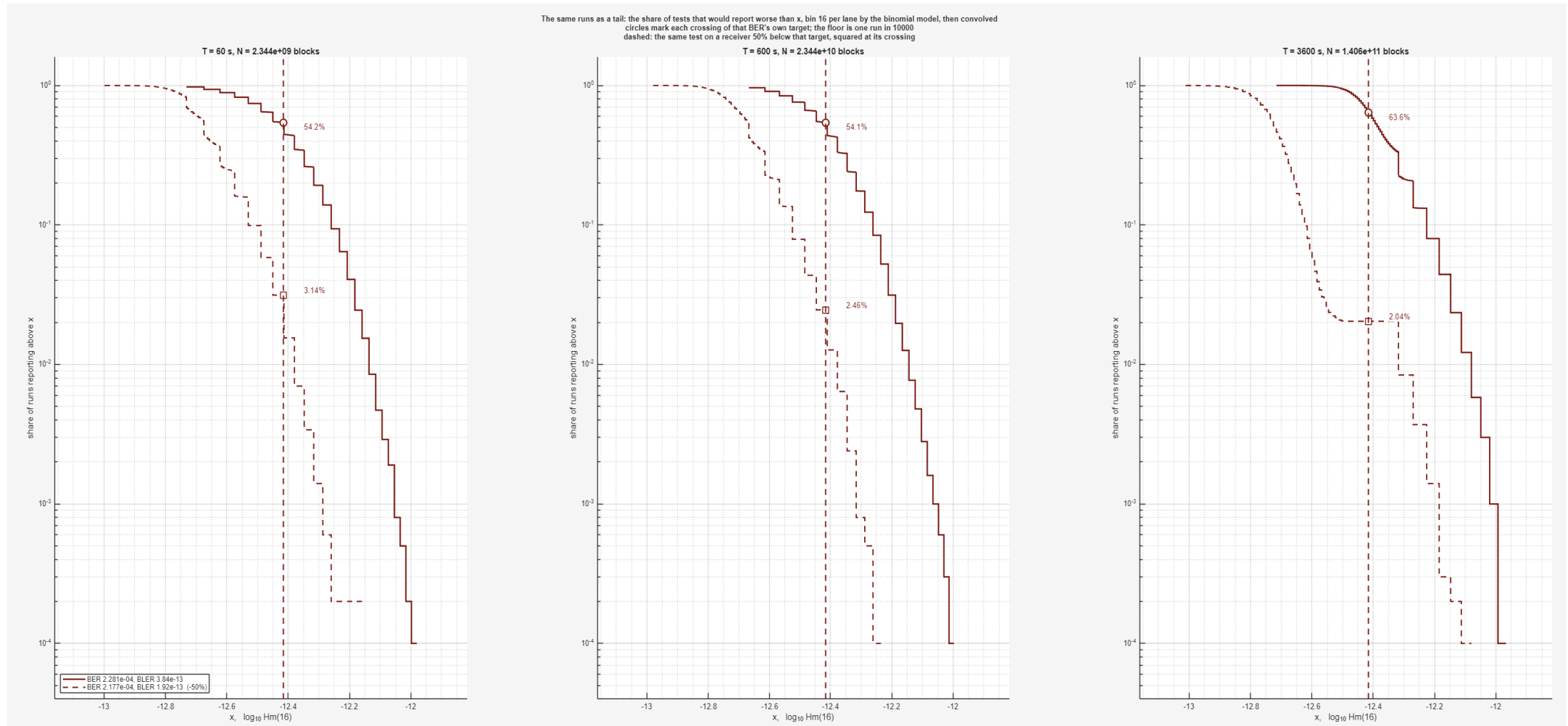
Proposed remedy binomial model distribution (1)

- Narrow scatter over random scenarios, based on Monte Carlo draws
- Reduces the probability for failure of marginal links (uncorrelated BER of $2.218\text{e-}4$), that should pass with 50%
- Secures that links with uncorrelated BER of $2\text{e-}4$ will pass, equivalent to $0.09[\text{dB}]/0.045[\text{dBo}]$ margin
- The figures below show the CCDF (Complementary CDF = probability of X value and above/worse) of the proposed extrapolation scheme for 1X links



Proposed remedy binomial model distribution (2)

- Very meaningful drop of the probability of false failure for links with BLER that is 50% lower ($1.92\text{e-}13$), equivalent to $0.032[\text{dB}]/0.016[\text{dBo}]$ margin
- The figures below show the CCDF (Complementary CDF = probability of X value and above/worse) of the proposed extrapolation scheme for 1X links



Proposed Remedy – Text

Replace the paragraph quoted from “180.9.14 Receiver sensitivity” with the following:

- NOTE—In order to predict whether a receiver meets the BLER requirement in a short test time, extrapolation of the measured histogram (see 174A.9.3) to $H_m(16)$ can be performed. This can be done by using binomial distribution model for uncorrelated errors as follows: For the highest bin having count of 10 or more, find which RS SER in equation 174A-5 would lead to that count from the total amount of FEC blocks. Then estimate $H_m(16)$ by accumulating equation 174A-5 with the found RS SER over bins 16 and above.

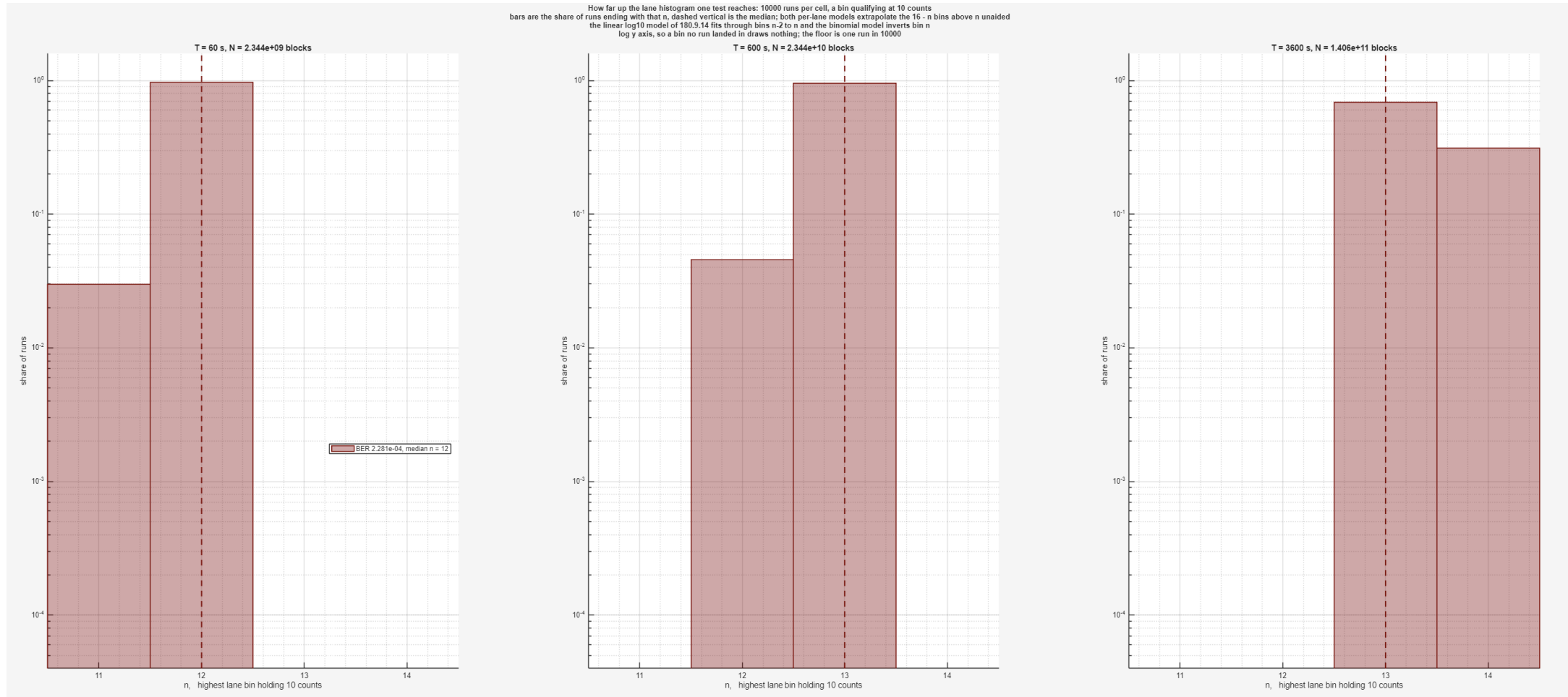
Alternatively, plug the target RS SER for the scenario into equation 174A-5 and see if the count of the highest bin having count of 10 or more is not higher than that value.

Backup Slides

180.9.14 log10 linear model distribution (3)

Explanation of step in the CCDF

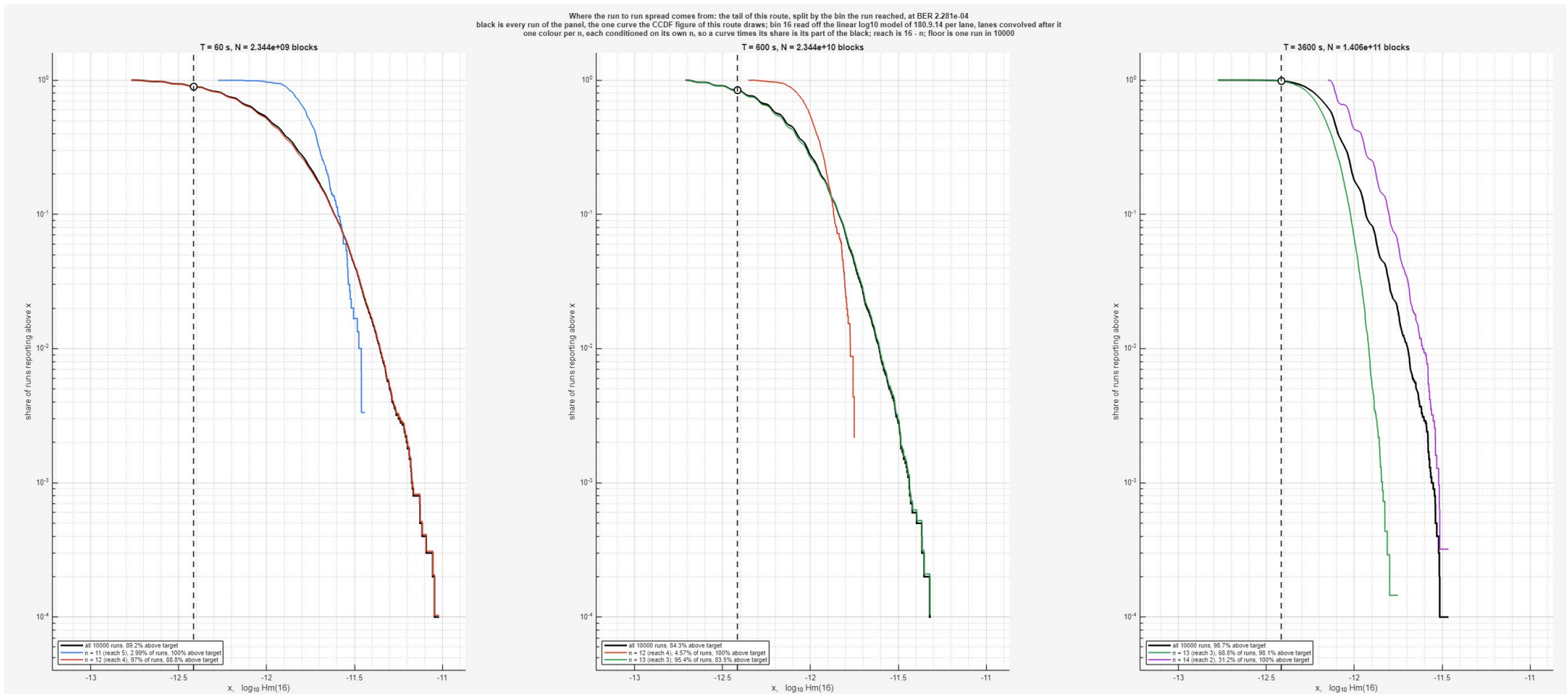
- The step in the CCDF (Complementary CDF = probability of X value and above/worse) for duration of 60[Sec] and for 600[Sec] results from having 2 values of max nonzero bins, where each of them has quite different bias.



180.9.14 log10 linear model distribution (4)

Explanation of step in the CCDF

- The step in the CCDF (Complementary CDF = probability of X value and above/worse) for duration of 60[Sec] and for 600[Sec] results from having dichotomic partitioning of the max nonzero bin being used, where each of the 2 max nonzero bin has quite different bias.

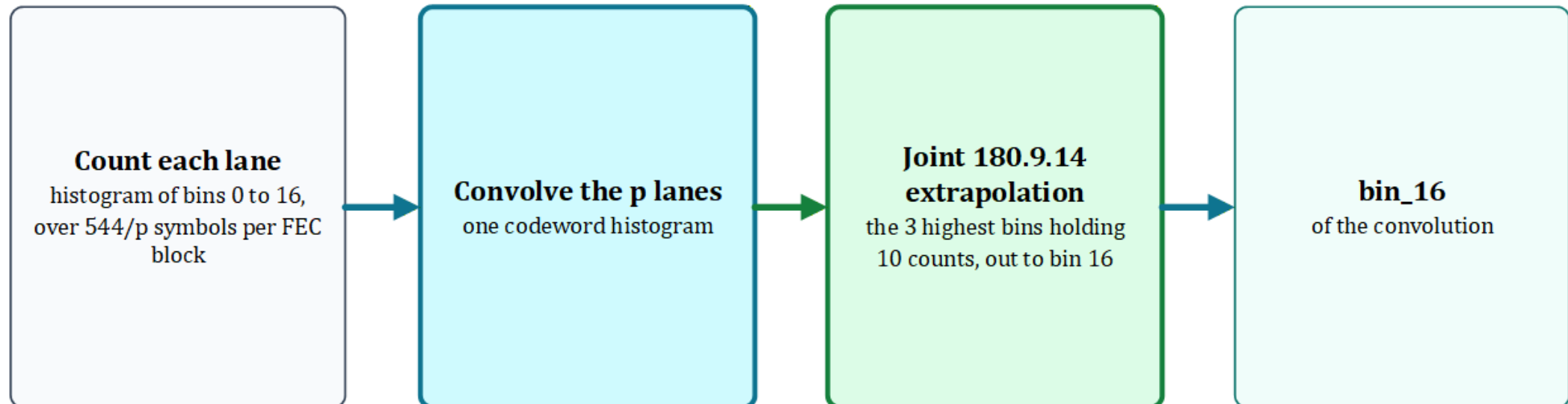


The Monte Carlo Analysis for P-lanes (1)

- Same flow as for 1X, only preceded by convolving the P-lanes bins values

p lanes: convolve first, extrapolate after

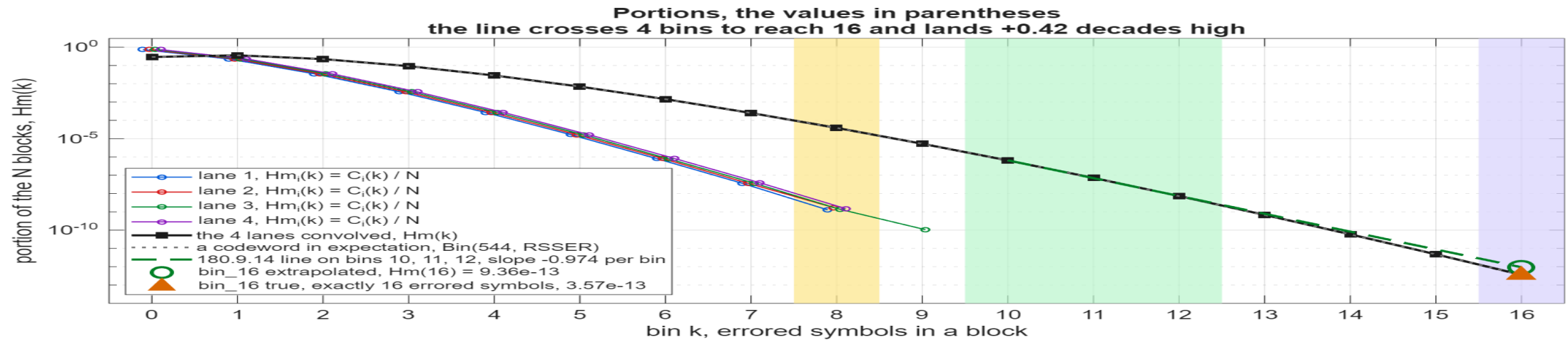
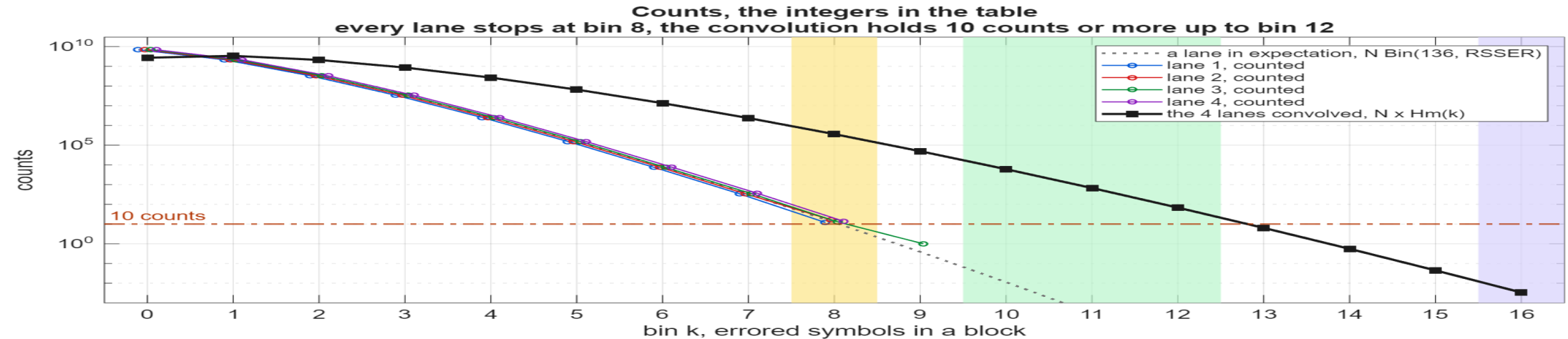
the limit is on the codeword, so there is nothing to extrapolate to until the lanes are one again



The Monte Carlo Analysis for P-lanes (1)

Numerical example – figure

One 60[Sec] run, $p = 4$, BER 2.281e-04: lanes counted, then convolved, then one 180.9.14 line to bin 16
 amber: the highest lane bin the 10 count rule allows. green: the three bins the line uses. violet: bin 16, exactly 16 errored symbols.
 the four lanes are drawn with a small horizontal offset, they lie on each other otherwise



The Monte Carlo Analysis for P-lanes (2)

Numerical example - table

One 60[Sec] run, $p = 4$, BER $2.281e-4$

the counts one draw put in every bin of every lane, the convolution of the four lanes, and the bin 16 the line gives
136 symbols per lane block, RSSER $2.279e-3$, $N = 9,375,000,000$ blocks per lane

bin	lane 1	lane 2	lane 3	lane 4	convolution
0	6,874,207,395 (0.733249)	6,874,167,746 (0.733245)	6,874,141,030 (0.733242)	6,874,076,591 (0.733235)	2,709,952,832 (0.289062)
1	2,135,296,074 (0.227765)	2,135,302,482 (0.227766)	2,135,337,976 (0.227769)	2,135,366,363 (0.227772)	3,367,184,898 (0.359166)
2	329,194,545 (0.0351141)	329,225,893 (0.0351174)	329,217,711 (0.0351166)	329,255,323 (0.0351206)	2,088,080,383 (0.222729)
3	33,589,038 (0.00358283)	33,592,635 (0.00358321)	33,590,620 (0.003583)	33,586,918 (0.0035826)	861,666,835 (0.0919111)
4	2,550,531 (0.000272057)	2,549,290 (0.000271924)	2,551,121 (0.00027212)	2,552,887 (0.000272308)	266,191,160 (0.0283937)
5	154,425 (1.6472e-5)	153,885 (1.64144e-5)	153,756 (1.64006e-5)	153,807 (1.64061e-5)	65,665,575 (0.00700433)
6	7,638 (8.1472e-7)	7,711 (8.22507e-7)	7,425 (7.92e-7)	7,737 (8.2528e-7)	13,474,055 (0.00143723)
7	342 (3.648e-8)	342 (3.648e-8)	347 (3.70133e-8)	360 (3.84e-8)	2,365,435 (0.000252313)
8	12 (1.28e-9)	16 (1.70667e-9)	13 (1.38667e-9)	14 (1.49333e-9)	362,699 (3.86879e-5)
9	0 (0)	0 (0)	1 (1.06667e-10)	0 (0)	49,349 (5.26393e-6)
10	0 (0)	0 (0)	0 (0)	0 (0)	6,034 (6.43574e-7)
11	0 (0)	0 (0)	0 (0)	0 (0)	670 (7.14277e-8)
12	0 (0)	0 (0)	0 (0)	0 (0)	68 (7.25707e-9)
13	0 (0)	0 (0)	0 (0)	0 (0)	6 (6.7968e-10)
14	0 (0)	0 (0)	0 (0)	0 (0)	1 (5.90088e-11)
15	0 (0)	0 (0)	0 (0)	0 (0)	0 (4.77142e-12)
16	0 (0)	0 (0)	0 (0)	0 (0)	0 (3.60634e-13)

The lanes reach bin 8

bin 8 holds 12, 16, 13, 14

bin 9 holds 0, 0, 1, 0

nothing above it

The convolution reaches 12

$N \times H_m(k) = 6034, 670, 68$

in bins 10, 11 and 12

bin 13 gives 6, under 10

One 180.9.14 line

$\log_{10} H_m(k)$ on those bins

slope -0.974 per bin

four bins out to 16

bin_16, $H_m(16) = 9.4e-13$

exactly 16 errored symbols

$N \times H_m(16) = 0.009$ counts

true $3.6e-13$, +0.42 decades

Portions, $H_m(k)$, in parentheses under every count, $N \times H_m(k)$

Lane i is counted, and its portion of the N blocks is $H_{m,i}(k) = C_i(k) / N$, so lane 1 at $k = 0$ holds $6,874,207,395 / 9,375,000,000 = 0.733249$.

The $p = 4$ lanes convolve into the codeword portion $H_m(k) = \text{SUM } H_{m,1}(k_1) H_{m,2}(k_2) H_{m,3}(k_3) H_{m,4}(k_4)$ over $k_1 + k_2 + k_3 + k_4 = k$, read back as $N \times H_m(k)$.

$k = 0$, the one route, all four lanes empty: $H_m(0) = 0.733249 \times 0.733245 \times 0.733242 \times 0.733235 = 0.289062$, $N \times H_m(0) = 2,709,952,832$.

$k = 1$, the four routes, the single error in lane i : $H_m(1) = 0.227765 \times 0.733245 \times 0.733242 \times 0.733235 + \text{the 3 alike} = 0.359166$, $N \times H_m(1) = 3,367,184,898$.

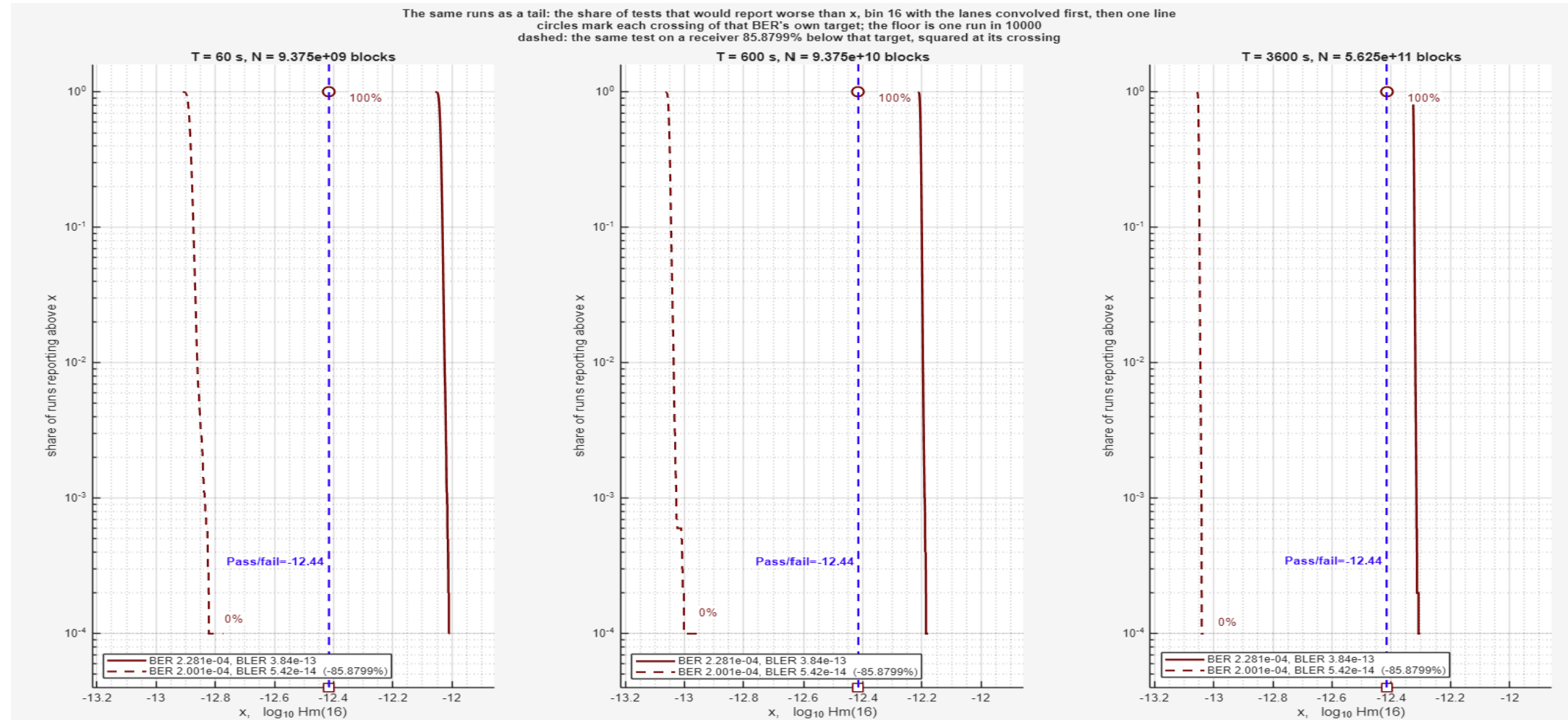
green: the bins the line uses. violet: bin 16, exactly 16 errored symbols, never populated. amber: the highest lane bin the 10 count rule allows.

For p-lanes: 180.9.14 log10 linear model distribution (3)

Extremely narrowed

Target BER=2.28e-4, leading to BLER=3.84e-13, vs. BER=2.0e-4 leading to BLER=5.42e-14 (-85.9%)

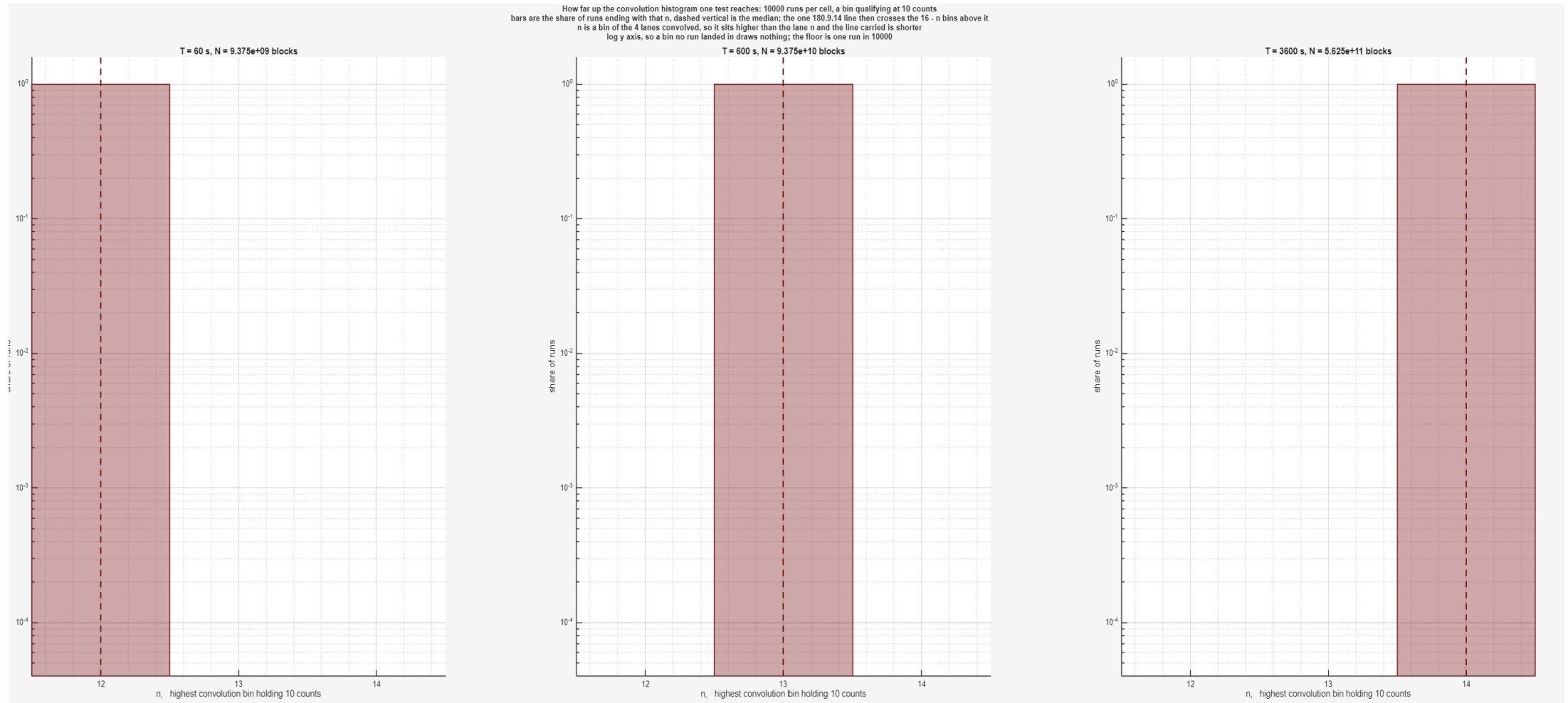
- The figure below shows that for 4-lanes there is much narrower scatter vs. per single lane
- Reason is that 100% of the cases get to same highest bin post convolution



For p-lanes: 180.9.14 log10 linear model distribution (4)

Target BER=2.28e-4, leading to BLER=3.84e-13, vs. BER=2.0e-4 leading to BLER=5.42e-14 (-85.9%)

- The figure below shows that for 4-lanes there is much narrower scatter vs. per single lane
- Reason is that post convolution, the highest bin is fixed over the 10,000 draws



Correlation impact on model bias (1)

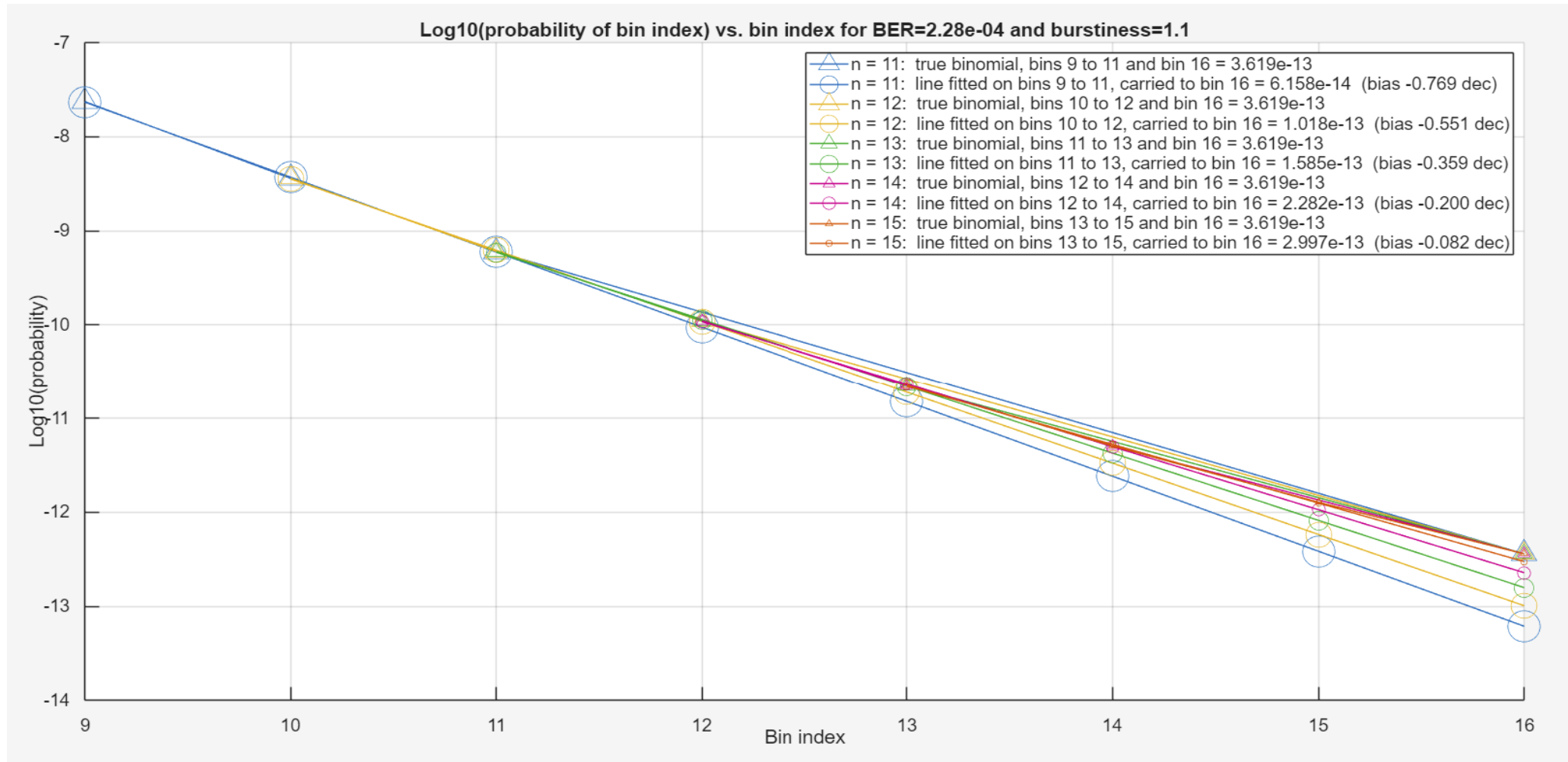
- As correlation of RS symbol errors increases the model bias is reduced
- With the used model* the correlation leading to negative bias (false pass) is only for failing links

Max bin >=10	Fixed Burstiness**=1.00		Fixed Burstiness**=1.05		Fixed Burstiness**=1.1	
	Extrapolated Bin 16	Model bias	Extrapolated Bin 16	Model bias	Extrapolated Bin 16	Model bias
11	1.57e-12	0.637	2.91e-13	-0.094	6.16e-14	-0.769
12	9.42e-13	0.415	2.95e-13	-0.088	1.02e-13	-0.551
13	6.42e-13	0.245	3.09e-13	-0.068	1.59e-13	-0.359
14	4.87e-13	0.129	3.27e-13	-0.043	2.28e-13	-0.200
15	4.05e-13	0.049	3.46e-13	-0.019	3.00e-13	-0.082

*Assuming Equivalent RSSER of 2.28e-3 for bin 16, and rising to this RSSER with fixed burstiness from the bins used for extrapolation

**Definition: Burstiness = Equivalent RSSER of bin (n) / Equivalent RSSER of bin (n – 1)

Correlation impact on model bias



*Assuming Equivalent RSSER of $2.28\text{e-}3$ for bin 16, rising to this RSSER with fixed burstiness from the lowest bin used for extrapolation

**Definition: Burstiness = Equivalent RSSER of bin (n) / Equivalent RSSER of bin ($n - 1$)