A Path toward Incorporating Advanced Signal Processing in Electrical Channel Performance Assessment - Recap

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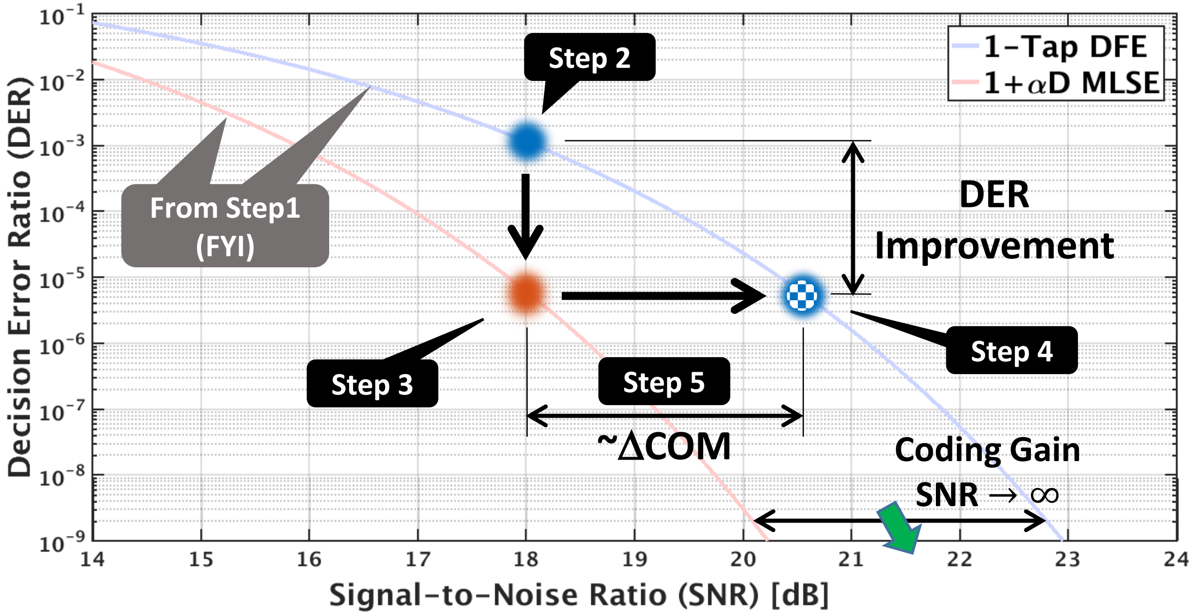
Introduction

- A method for incorporating performance effect of MLSE in the COM flow was proposed in "shakiba_3df_01b_2211.pdf" and further explained in "shakiba_3dj_01_230116.pdf"
- This presentation is a recap
- More time is dedicated to Q&A
- Minor updates are included based on the feedback
- More analysis and extensions will be presented in future meetings, specifically on:
  - Noise coloring
  - Error propagation (comparisson to DFE, pre-FEC statistics, and post-FEC expectations)
  - Pre-coding
- Thanks to Rich for COM 4.0 Matlab code update (Mellitz_3dj_elect_01_230223.pdf)
- How to use the MLSE performance advantage along with margining are subjects for further discussions

Proposal Recap

- The proposal outlined a method that enables incorporating MLSE impact in the COM flow
- As rates increase, we see this as a useful and maybe necessary update to COM for reflecting a more accurate and realistic representation of capabilities in the reference receiver
- Rx FFE support is essential to considering more advanced detection techniques than DFE
- $1+\alpha D$ MLSE appears as a first natural alternative to a 1-tap DFE
- Proposal specified following steps:
  1) Use COM analysis to find the DFE tap, α
  2) Use analysis to calculate SNR_{DFE} and DER_{DFE}
  3) Use analysis to calculate DER_{MLSE} at SNR_{DFE}
  4) Use analysis to calculate $SNR_{DFE,equivalent}$ for the same DFE that yields the same DER_{MLSE}
  5) Increase from SNR_{DFE} to $SNR_{DFE,equivalent}$ is a much better estimate of COM improvement (ΔCOM) offered by the MLSE than $10\log_{10}(1+\alpha^2)$

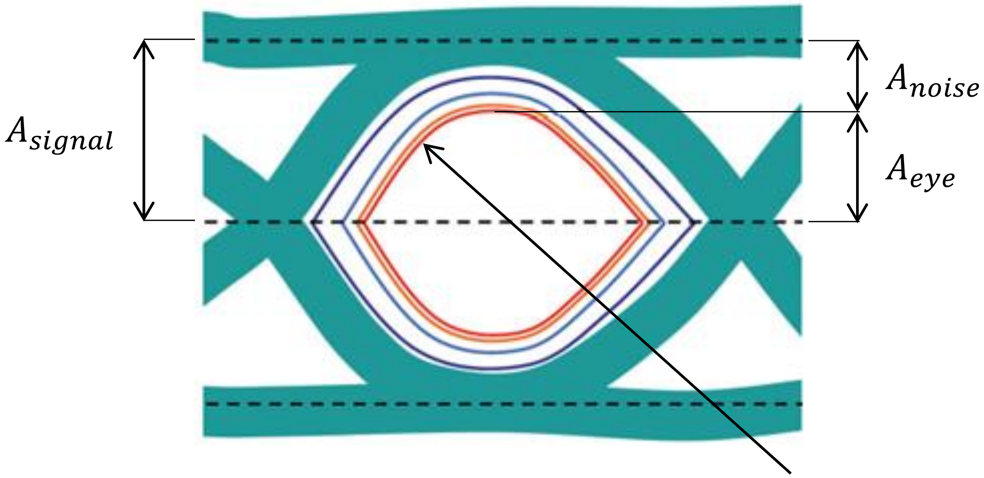


SNR, COM, and VEC

$$COM = 20\log_{10}\left(\frac{A_{signal}}{A_{noise}}\right)$$

$$VEC = 20\log_{10}\left(\frac{A_{signal}}{A_{eye}}\right)$$

$\rightarrow \quad COM = -20\log_{10}\left(1 - 10^{-VEC/20}\right)$



- COM and VEC are related to SNR

$$SNR[dB] = 10\log_{10}\left(\frac{1}{3}\frac{L+1}{L-1}\frac{{A_{peak}}^2}{{\sigma_{noise}}^2}\right)$$ (Appendix A)

$$A_{peak} = (L-1)A_{signal}$$

$$A_{noise} = k_{DER}\sigma_{noise}$$ ← k_{DER} is a multiplier factor that determines how many σ's away from mean achieves target DER (a.k.a. Q factor for Gaussian noise)

- As a result COM can be expressed as

$$COM = SNR[dB] - 10\log_{10}\left(\frac{L^2-1}{3}{k_{DER}}^2\right)$$

- Which suggests that COM is in fact a kind of SNR with a notion of DER directly built in it

SNR, COM, and VEC

$$COM = SNR[dB] - 10\log_{10}\left(\frac{L^2-1}{3}{k_{DER}}^2\right)$$

- There are three ways to interpret this equation
  1) If after a change in SNR same DER is targeted, k_{DER} remains constant and any increase (decrease) in SNR translates to the same increase (or decrease) in COM

$$\boxed{\Delta COM = \Delta SNR[dB]}$$

     e.g. this means that if a 3dB COM is targeted and if MLSE is used in the actual receiver with a verified 1.8dB SNR gain, COM target with the reference receiver can be lowered to 1.2dB.

  2) If after a change in SNR same COM is targeted, COM remains constant and any increase (or decrease) in SNR translates to an increase (or decrease) in k_{DER} and results in a decrease (or increase) in DER
  3) A change in SNR can be broken down to partially achieve both above as long as the equation holds
- Any change in SNR also translates to a change in VEC, but the change depends on VEC value

$$\Delta VEC = \Delta SNR[dB] - 20\log_{10}\left(\left(10^{\Delta SNR[dB]/20} - 1\right)10^{VEC/20} + 1\right)$$

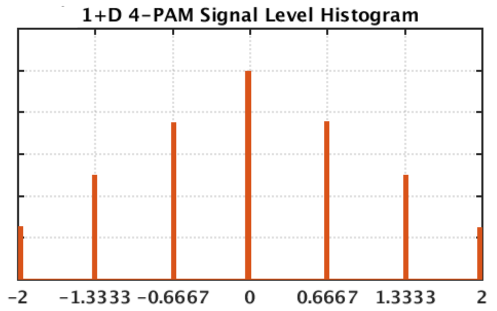
- Note that if ΔSNR > 0, ΔVEC is negative, showing improvement in VEC

Comments on Rx FFE

- An Rx FFE is added after CTLE to optimally shape the pulse response to only one post-cursor
- There is a current discussion on whether FFE can continue to be mimicked with a DFE to simplify the COM flow and avoid experienced issues with optimizing FFE coefficients
- FFE is the right choice
  - Signal equalization quality is different between FFE and DFE (particularly FFE can handle pre-cursors too)
  - Noise handling is different between FFE and DFE
  - FFE is closer to real implementations
  - As links become more challenging, margins shrink and more realistic models help more
  - FFE keeps the door open to further signal processing (DFE makes hard decisions and terminates signal chain)
- This is not to say that using DFE is impossible, but not without compromises, for example
    - Will there be consensus on the quantification and mitigation of performance difference?
    - There will be no more pre-cursor equalization than what is available now
    - What is the impact on extending to higher-order MLSEs?
- MLSE gain is on top of the gain that Rx FFE provides, so with or without FFE, there is still a gain

Coding View of MLSE

- The entire end-to-end signal path response (e.g. Tx FFE + Tx Filters + Channel + Rx Filters + CTLE + Rx FFE) is optimally equalized to $1+\alpha D$ (historically, α would be the DFE tap)
- $1+\alpha D$ increases the number of signal levels
  - e.g. increase 4 levels of PAM4 (±1, $\pm1/3$) to generally 16 levels ($\pm1\pm\alpha$, $\pm1\pm\alpha/3$, $\pm1/3\pm\alpha$, $\pm1/3\pm\alpha/3$)
- Depending on α, some of the levels may merge and become not equally-likely
  - e.g. for α=1, 4 levels of PAM4 (±1, $\pm1/3$) become 7 levels (±2, $\pm4/3$, $\pm2/3$, 0), with smaller levels being more likely

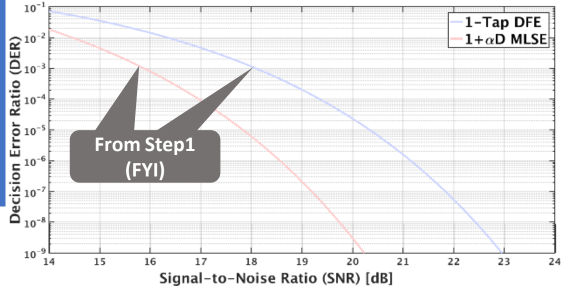


- $1+\alpha D$ is a level-coding operation since levels become correlated
  - Not all level transitions in a sequence of transitions are allowed (redundancy)
  - e.g. level $1+\alpha$ can only transition to one of 4 levels $\pm1+\alpha$ and $\pm1/3+\alpha$
  - Illegal level transitions can be detected and with most-likelihood criteria corrected
- $1+\alpha D$ is the simplest form of Partial Response Signaling (PRS), aka correlative level coding
- Essentially it can act as a FEC!
  - In amplitude domain, not time
  - No increase data rate and amplitude damage is done regardless

Coding View of MLSE

- Compare the two following options:
  1) Use a 1-tap DFE
     - DFE removes first cursor ISI (αD term in the 1+αD expression), ignoring and wasting the redundancy
  2) Use a 1+αD MLSE
     - Sequence processing leverages correlation between levels to detect illegal level transitions (error detection) and provides best estimate to the sequence it believes was most likely transmitted (error correction)
     - Encoding is already done for free, just need to decode (likely more efficient than moving to a stronger FEC)
- Similar to any other coding scheme (such as RS FEC), the advantage can be expressed by a coding gain (usually in dB)
- Coding gain of the 1+αD MLSE scheme is $10\log_{10}(1+\alpha^2)$
- Coding gain can only be achieved asymptotically and is agnostic to potentially-important particular link characteristics and operating conditions such as noise distributions and DER
- Our proposal provides a simple and COM-compatible way of calculating the real advantage based on the actual link characteristics and conditions
- The idea and proposal can be extended to higher-order PRS polynomials (e.g. $1+\alpha D+\beta D^2$)

Explanation of Steps (Step 1)

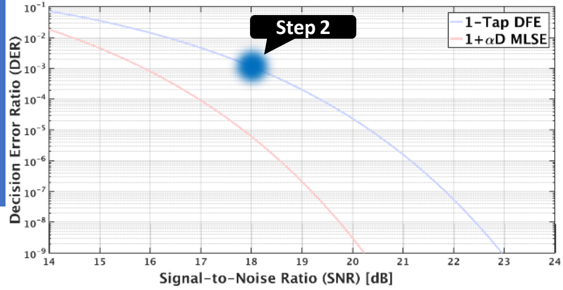


1) Determining the optimum DFE tap is a standard practice in COM

$$\boxed{\alpha = DFE\ Tap}$$

- For optimization, it is possible, and maybe recommended to skip the DFE and optimize α directly for best MLSE performance
- This can be simply achieved by changing the signal energy in the COM-defined FOM to include energy of the post-cursor (MLSE treats the post-curser as a part of the signal)
- The MLSE-based optimization has not been implemented here:
  - To provide a more direct and side-by-side comparison
  - The additional performance improvement is not typically expected to be considerable
  - To keep the existing COM flow untouched

Explanation of Steps (Step 2)



2) Equation to calculate SNR_{DFE} for L-PAM (Appendix A)

$$\boxed{SNR_{DFE} = \frac{1}{3}\frac{L+1}{L-1}\frac{main^2}{\sigma_{noise}{}^2}}$$

← Note that this can be written as →

$$\boxed{\frac{main}{L-1} = \sigma_{noise}\sqrt{\frac{3}{L^2-1}SNR_{DFE}}}$$

$main =$ Main cursor of the pulse response at the Rx FFE output

- Pulse amplitude = Peak value of the transmitted ± PAM signal swing
- Calculating main cursor is a standard practice in COM

$\sigma_{noise} =$ Standard deviation of the total noise (Xtalk, TX SNR, Rx eta0, Jitter, and ISI) at the Rx FFE output

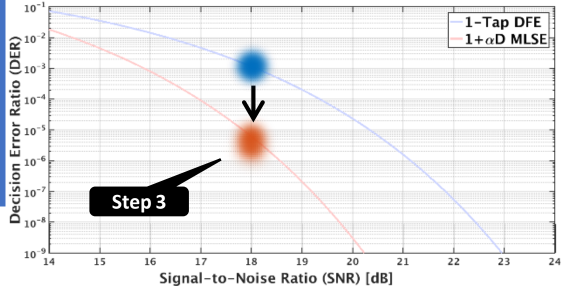
- Calculated from the total noise PDF
- Calculating the total noise PDF is a standard practice in COM

Equation to calculate DER_{DFE} for L-PAM (Appendix B)

$$DER_{DFE} \approx \frac{2}{\frac{1}{L-1}+CDF_{noise}\left((1-2\alpha)\frac{main}{L-1}\right)}\left(1 - CDF_{noise}\left(\frac{main}{L-1}\right)\right)$$

- The above equation includes effect of DFE error propagation
- Calculating the total noise CDF is a standard practice in COM
- Note that DER_{DFE} is not needed for obtaining ΔCOM, but is still useful to calculate the decrease in error ratio

Explanation of Steps (Step 3)

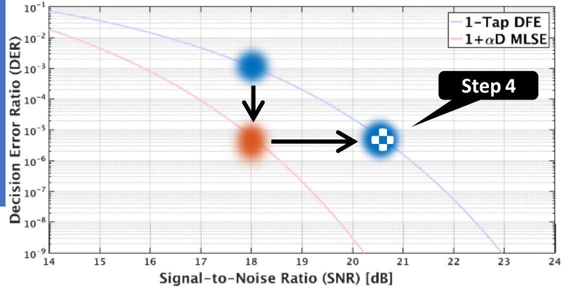


3) Equation to calculate DER_{MLSE} for L-PAM (Appendix C)

$$\boxed{DER_{MLSE} \approx 2\sum_{j=1}^{\infty} j\left(\frac{L-1}{L}\right)^{j}\left(1 - CDF_{noise}\left(\sqrt{1+(j-1)(1-\alpha)^2+\alpha^2}\,\frac{main}{L-1}\right)\right)}$$

- The above equation includes effect of MLSE error propagation
- The summation is expected to include enough terms so that adding more terms doesn't considerably change the result anymore
- Calculating the total noise CDF is a standard practice in COM

Explanation of Steps (Step 4)



4) Equation to calculate $\mathrm{SNR}_{\mathrm{DFE,equivalent}}$ (Appendix D)

$$\boxed{SNR_{DFE,equivalent} \approx \left(\frac{L-1}{main} CDF_{noise}^{-1}\left(1-\frac{1}{2}DER_{MLSE}\left(\frac{1}{L-1}+CDF_{noise}\left((1-2\alpha)\frac{main}{L-1}\right)\right)\right)\right)^2 SNR_{DFE}}$$

$CDF_{noise}^{-1} =$ Inverse function of the total noise CDF

- Calculating the total noise CDF (hence inverse CDF) is a standard practice in COM

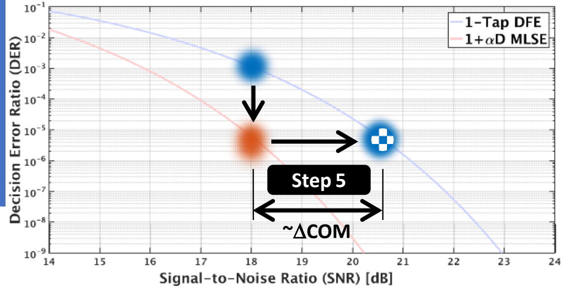
- The above equation includes effect of MLSE error propagation

Equation to calculate $\sigma_{\mathrm{noise,equivalent}}$ (Appendix D)

$$\sigma_{noise,equivalent} \approx \frac{1}{\frac{L-1}{main}CDF_{noise}^{-1}\left(1-\frac{1}{2}DER_{MLSE}\left(\frac{1}{L-1}+CDF_{noise}\left((1-2\alpha)\frac{main}{L-1}\right)\right)\right)}\sigma_{noise}$$

- The above equation alternatively suggests that the noise PDF and CDF can be horizontally scaled by the given factor to obtain PDF and CDF of the equivalent noise
- Note that calculating $\sigma_{\mathrm{noise,equivalent}}$ is not necessary and is only an alternative to calculating $SNR_{DFE,equivalent}$

Explanation of Steps (Step 5)



5) Equation to calculate increase in SNR

$$\frac{SNR_{DFE,equivalent}}{SNR_{DFE}} \approx \left(\frac{L-1}{main} CDF_{noise}^{-1}\left(1-\frac{1}{2} DER_{MLSE}\left(\frac{1}{L-1}+CDF_{noise}\left((1-2\alpha)\frac{main}{L-1}\right)\right)\right)\right)^2$$

Equation to calculate equivalent decrease in noise

$$\frac{\sigma_{noise}}{\sigma_{noise,equivalent}} \approx \frac{L-1}{main} CDF_{noise}^{-1}\left(1-\frac{1}{2} DER_{MLSE}\left(\frac{1}{L-1}+CDF_{noise}\left((1-2\alpha)\frac{main}{L-1}\right)\right)\right)$$

Equation to calculate ΔCOM

$$\boxed{\Delta COM \approx 10\log_{10}\left(\frac{SNR_{DFE,equivalent}}{SNR_{DFE}}\right)} = 20\log_{10}\left(\frac{\sigma_{noise}}{\sigma_{noise,equivalent}}\right)$$

Equation to calculate reduction in DER

$$Reduction\ in\ DER \approx \frac{DER_{MLSE}}{DER_{DFE}}$$

Recap (4-PAM, L = 4)

$$DER_{MLSE} \approx 2\sum_{l=1}^{\infty} l\left(\frac{L-1}{L}\right)^{l} Q\left(\frac{\sqrt{1+(i-1)(1-\alpha)^2+\alpha^2}}{(L-1)\sigma_{total\_noise}}\right)$$

$$DER_{DFE} \approx \frac{2}{\frac{L}{L-1} - Q\left(\frac{1-2\alpha}{(L-1)\sigma_{total\_noise}}\right)} Q\left(\frac{1}{(L-1)\sigma_{total\_noise}}\right)$$

$$SNR_{DFE} = \frac{1}{3}\frac{L+1}{L-1} \times \frac{main^2}{\sigma_{noise}{}^2} \quad \leftrightarrow \quad \frac{main}{L-1} = \sigma_{noise}\sqrt{\frac{3}{L^2-1} SNR_{DFE}}$$

2

$$DER_{DFE} \approx \frac{2}{\frac{1}{L-1} + CDF_{noise}\left((1-2\alpha)\frac{main}{L-1}\right)}\left(1 - CDF_{noise}\left(\frac{main}{L-1}\right)\right)$$

2

$$DER_{MLSE} \approx 2\sum_{j=1}^{\infty} j\left(\frac{L-1}{L}\right)^{j}\left(1 - CDF_{noise}\left(\sqrt{1+(j-1)(1-\alpha)^2+\alpha^2}\,\frac{main}{L-1}\right)\right)$$

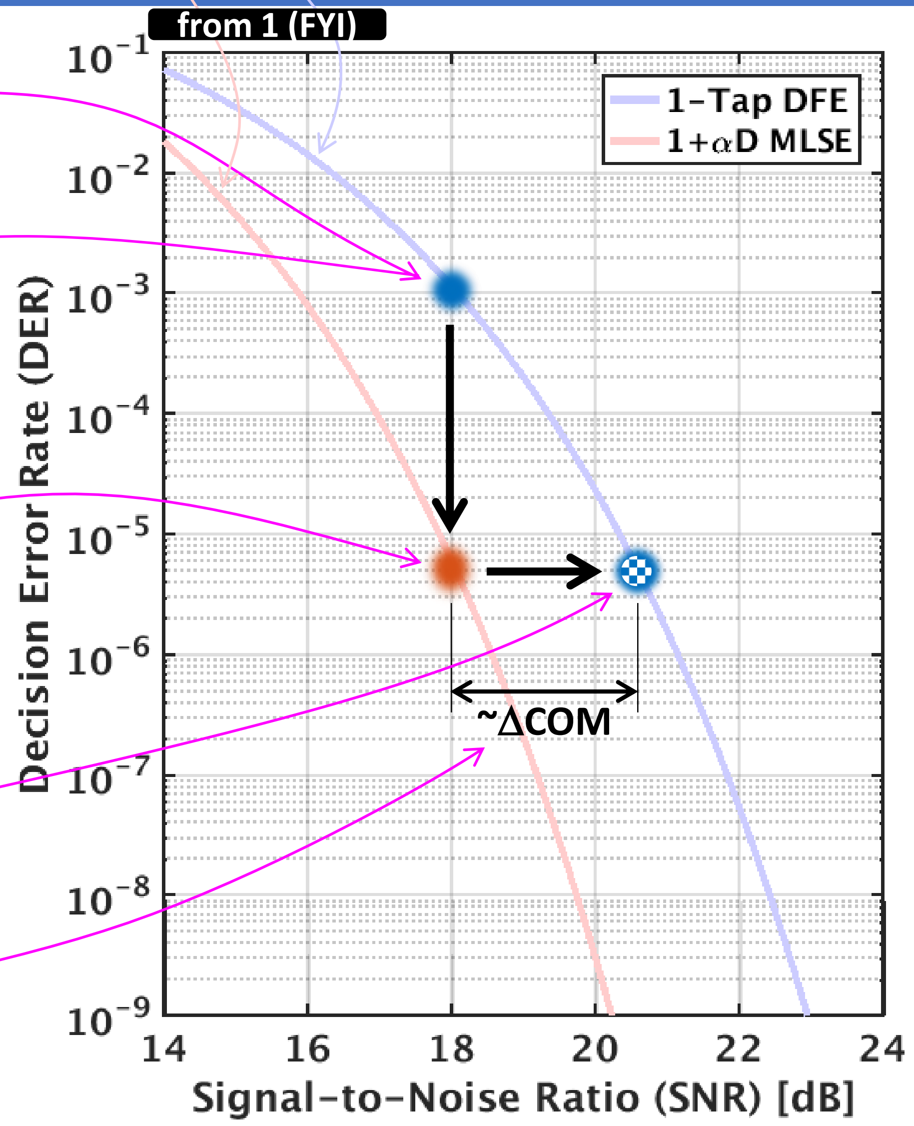
3

$$SNR_{DFE,equivalent} \approx \left(\frac{L-1}{main} CDF_{noise}^{-1}\left(1 - \frac{1}{2} DER_{MLSE}\left(\frac{1}{L-1} + CDF_{noise}\left((1-2\alpha)\frac{main}{L-1}\right)\right)\right)\right)^2 SNR_{DFE}$$

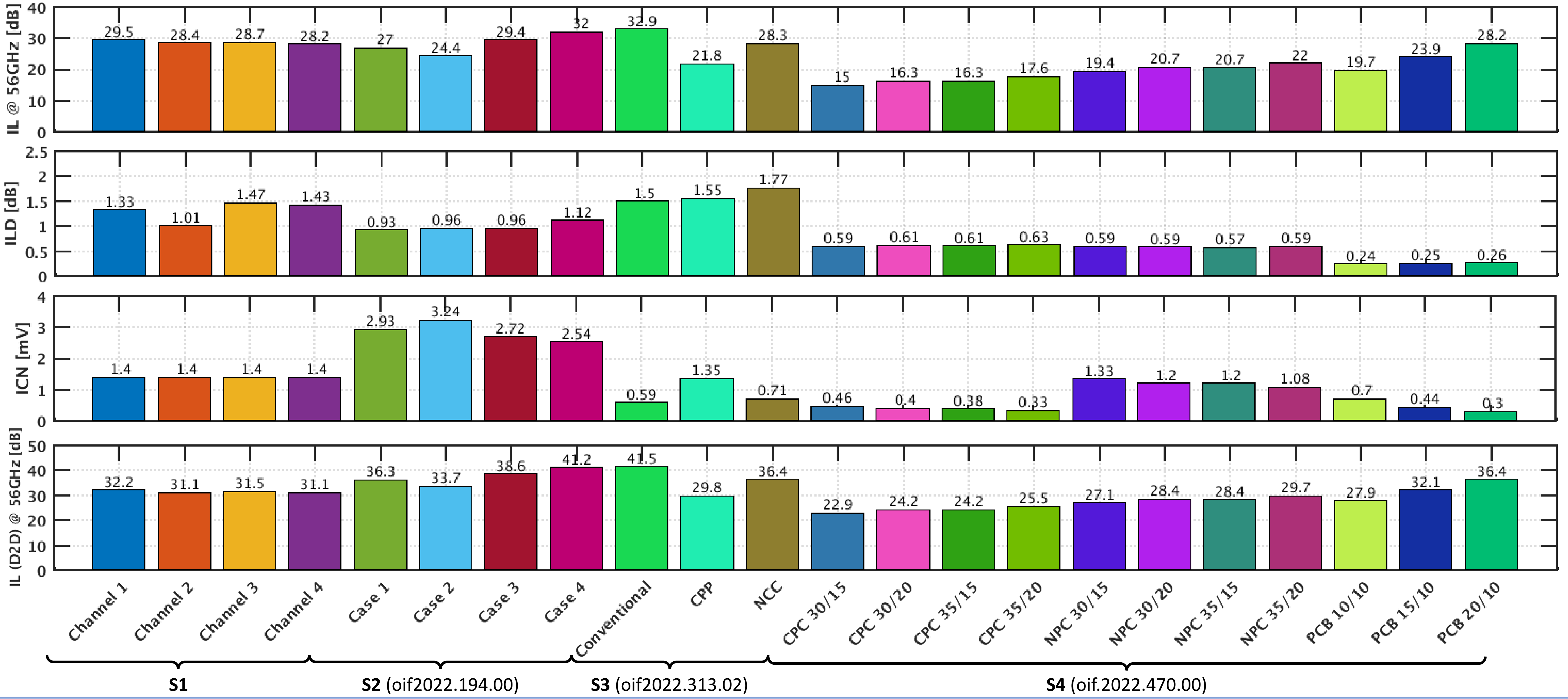
4

$$\Delta COM \approx 10\log_{10}\left(\frac{SNR_{DFE,equivalent}}{SNR_{DFE}}\right)$$

5



Study Cases



IL @ 56GHz [dB]
29.5
28.4
28.7
28.2
27
24.4
29.4
32
32.9
21.8
28.3
15
16.3
16.3
17.6
19.4
20.7
20.7
22
19.7
23.9
28.2
ILD [dB]
1.33
1.01
1.47
1.43
0.93
0.96
0.96
1.12
1.5
1.55
1.77
0.59
0.61
0.61
0.63
0.59
0.59
0.57
0.59
0.24
0.25
0.26
ICN [mV]
1.4
1.4
1.4
1.4
2.93
3.24
2.72
2.54
0.59
1.35
0.71
0.46
0.4
0.38
0.33
1.33
1.2
1.2
1.08
0.7
0.44
0.3
IL (D2D) @ 56GHz [dB]
32.2
31.1
31.5
31.1
36.3
33.7
38.6
41.2
41.5
29.8
36.4
22.9
24.2
24.2
25.5
27.1
28.4
28.4
29.7
27.9
32.1
36.4
Channel 1
Channel 2
Channel 3
Channel 4
Case 1
Case 2
Case 3
Case 4
Conventional
CPP
NCC
CPC 30/15
CPC 30/20
CPC 35/15
CPC 35/20
NPC 30/15
NPC 30/20
NPC 35/15
NPC 35/20
PCB 10/10
PCB 15/10
PCB 20/10
S1
S2 (oif2022.194.00)
S3 (oif2022.313.02)
S4 (oif.2022.470.00)

Link Parameters

Channel	Bit Rate [Gb/s]	Thru Swing [mV]	Fext Swing [mV]	Next Swing [mV]	TX FIR [Pre / Post]	Die C_d [fF] L_s [pH]	C_b [fF]	Package [mm] [Ω]	Rx Filter BW	CTLE Pole/Zero Ratio	DFE [# of Taps]	Rx FFE [Pre / Post]	TX SNR [dB]	Rx Noise [V^2/GHz]	Jitter Rand / DD [UI]	k_N *		
																Native	1E-3	1E-4
S1	224	413	413 x k_N	608 x k_N	3 / 1	40/90/110 130/150/140	Included In channel	Included In channel	0.75 x f_b	80/2.5/1	1	6 / 8	32.5 - $20\log_{10}(k_N)$	4.1E-8 x k_N^2	0.01 / 0.02 x k_N	1	2	1.675
																1	2.05	1.725
																1	1.9	1.575
																1	2	1.675
S2	224	442	442 x k_N	608 x k_N	3 / 1	40/90/110 130/150/140	30	30 92.5	0.75 x f_b	100/2.5/1	1	0 / 24	33 - $20\log_{10}(k_N)$	4.1E-8 x k_N^2	0.01 / 0.02 x k_N	1	1.85	1.6
																1	2	1.725
																1	1.7	1.425
																1	1.45	1.225
S3	224	413	413 x k_N	608 x k_N	3 / 1	40/90/110 130/150/140	30	30 92.5	0.75 x f_b	80/2.5/1	1	0 / 24	33 - $20\log_{10}(k_N)$	4.1E-8 x k_N^2	0.01 / 0.02 x k_N	1	0.9	0.735
																1	1.45	1.175
																1	0.95	0.775
S4	224	413	413 x k_N	608 x k_N	3 / 1	40/90/110 130/150/140	40	30 92.5	0.75 x f_b	80/2.5/1	1	0 / 24	33 - $20\log_{10}(k_N)$	4.1E-8 x k_N^2	0.01 / 0.02 x k_N	1	2.34	2
																1	2.25	1.9
																1	2.225	1.9
																1	2.1	1.775
																1	1.68	1.405
																1	1.525	1.275
																1	1.525	1.275
																1	1.41	1.175
																1	1.85	1.555
																1	1.46	1.235
																1	1.14	0.945

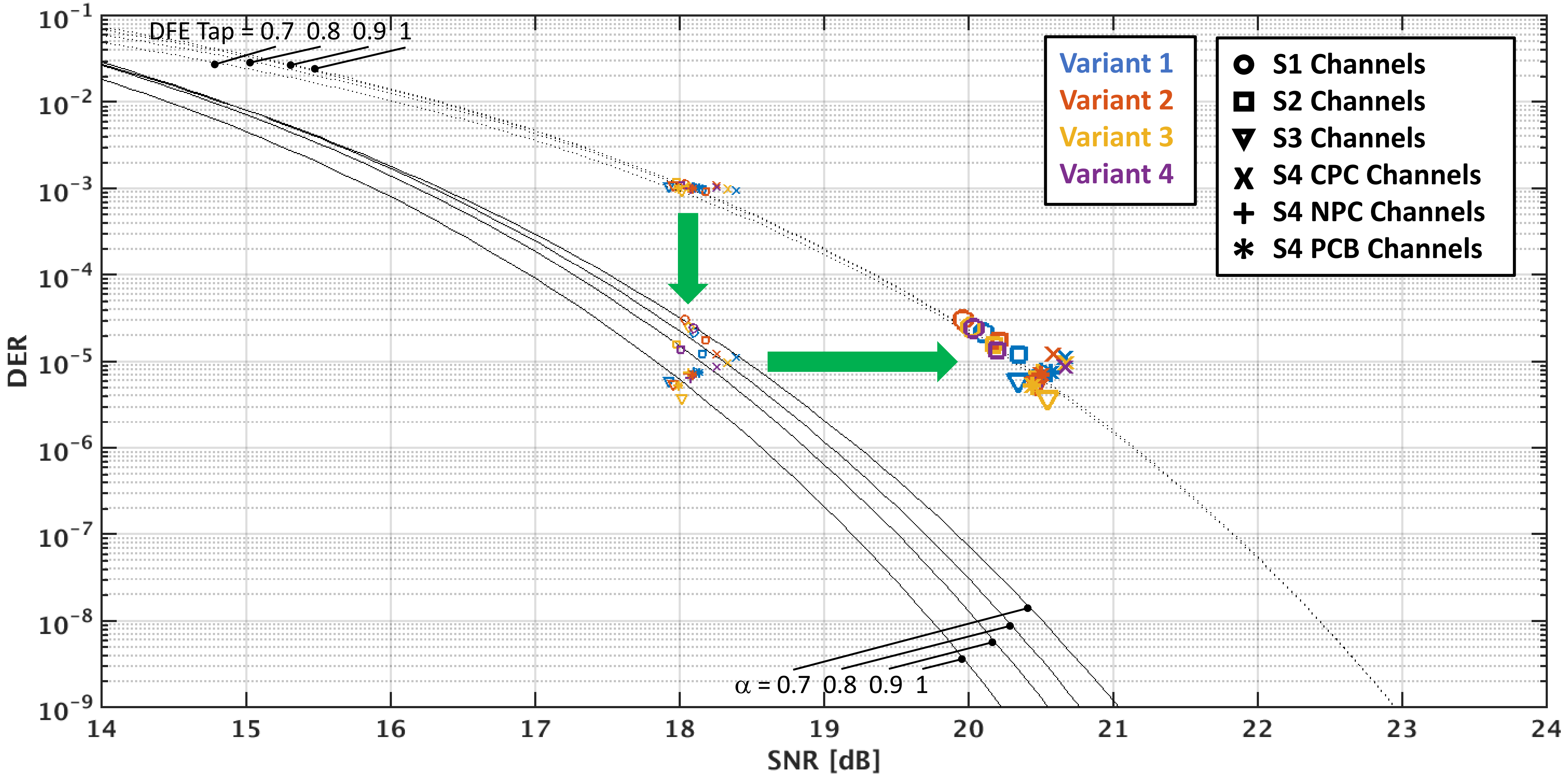
* To force more errors to facilitate time-domain simulation verifications

Summary of the Case Results (1E-3)

Channel	Variant	DFE Tap $= \alpha$	Theoretical Coding Gain [dB]	SNR_{DFE} [dB]	DER_{DFE}	DER_{MLSE}	$SNR_{DFE,equivalent}$ [dB]	Nosie Scaling Factor		$\Delta SNR = \Delta COM$ [dB]	DER Ratio [Order of Magnitude]	DER Simulation Results *		
								Total	for Simulation			DFE	MLSE	$DFE_{Equivalent}$
S1	Channel 1	**0.8121**	2.1997	18.0966	**1.0292 E-3**	**2.1588 E-5**	20.1022	**0.7938**	0.7465	**2.0055**	1.6783	1.04 E-3	2.2 E-5	2.5 E-5
	Channel 2	**0.7729**	2.0341	18.0369	**1.1216 E-3**	**3.0795 E-5**	19.9627	**0.8011**	0.7670	**1.9258**	1.5614	1.95 E-3	2.6 E-5	3.6 E-5
	Channel 3	**0.7778**	2.0546	18.0641	**1.0299 E-3**	**2.4942 E-5**	20.0097	**0.7993**	0.7467	**1.9456**	1.6159	5.5 E-4	1.4 E-5	3.3 E-5
	Channel 4	**0.7761**	2.0476	18.0914	**1.0044 E-3**	**2.4560 E-5**	20.0336	**0.7996**	0.7586	**1.9422**	1.6117	1.02 E-3	1.2 E-5	2.4 E-5
S2	Case 1	**0.9025**	2.5876	18.1565	**9.6653 E-4**	**1.2347 E-5**	20.3399	**0.7777**	0.7552	**2.1833**	1.8937	1.71 E-3	3.3 E-5	3.6 E-5
	Case 2	**0.8219**	2.2417	18.1782	**9.3605 E-4**	**1.7755 E-5**	20.2069	**0.7917**	0.7788	**2.0288**	1.7220	1.42 E-3	7.1 E-5	4.1 E-5
	Case 3	**0.9119**	2.6283	17.9732	**1.1996 E-3**	**1.6031 E-5**	20.1700	**0.7765**	0.7417	**2.1968**	1.8741	2.73 E-3	7.2 E-5	6.8 E-5
	Case 4	**0.9026**	2.5880	18.0037	**1.1001 E-3**	**1.3840 E-5**	20.1903	**0.7774**	0.7321	**2.1866**	1.9002	2.56 E-3	9 E-6	3.6 E-5
S3	Conventional	**0.9672**	2.8680	17.9234	**1.0536 E-3**	**5.9785 E-6**	20.3362	**0.7575**	0.6115	**2.4128**	2.2461	1.94 E-3	1.9 E-5	5 E-6
	CPP	**0.9938**	2.9833	17.9546	**1.0985 E-3**	**5.4707 E-6**	20.4631	**0.7492**	0.5976	**2.5085**	2.3028	1.06 E-3	1 E-6	5 E-6
	NCC	**0.9933**	2.9812	18.0133	**9.5421 E-4**	**3.8148 E-6**	20.5386	**0.7477**	0.5601	**2.5253**	2.3982	1.40 E-3	1.9 E-5	5 E-6
S4	CPC 30/15	**0.9421**	2.7590	18.3817	**9.6187 E-4**	**1.1310 E-5**	20.6573	**0.7695**	0.7467	**2.2756**	1.9297	6.4 E-4	NA	0
	CPC 30/20	**0.9598**	2.8358	18.2483	**1.0965 E-3**	**1.2076 E-5**	20.5759	**0.7649**	0.7421	**2.3276**	1.9581	6.8 E-4	NA	8 E-6
	CPC 35/15	**0.9602**	2.8376	18.3260	**9.7801 E-4**	**9.6982 E-6**	20.6651	**0.7639**	0.7402	**2.3391**	2.0037	9.4 E-4	NA	2.4 E-5
	CPC 35/20	**0.9783**	2.9160	18.2488	**1.0360 E-3**	**8.7069 E-6**	20.6576	**0.7578**	0.7332	**2.4088**	2.0755	1.12 E-3	NA	6 E-6
	NPC 30/15	**0.9739**	2.8969	18.0914	**1.0265 E-3**	**7.6231 E-6**	20.4955	**0.7582**	0.7034	**2.4041**	2.1292	1.29 E-3	NA	5 E-6
	NPC 30/20	**0.9779**	2.9144	18.0586	**1.0508 E-3**	**7.3813 E-6**	20.4809	**0.7566**	0.6932	**2.4224**	2.1534	9.3 E-4	NA	5 E-6
	NPC 35/15	**0.9782**	2.9157	18.0532	**1.0596 E-3**	**7.4717 E-6**	20.4763	**0.7566**	0.6927	**2.4231**	2.1517	1.95 E-3	NA	8 E-6
	NPC 35/20	**0.9784**	2.9166	18.0672	**1.0116 E-3**	**6.4992 E-6**	20.4996	**0.7557**	0.6786	**2.4324**	2.1922	2.38 E-3	NA	1 E-6
	PCB 10/10	**0.9822**	2.9329	18.1274	**1.0372 E-3**	**7.5393 E-6**	20.5601	**0.7557**	0.7116	**2.4327**	2.1385	1.06 E-3	NA	2 E-5
	PCB 15/10	**0.9742**	2.8982	18.0862	**1.0108 E-3**	**7.1265 E-6**	20.4962	**0.7577**	0.7130	**2.4100**	2.1518	1.72 E-3	NA	4 E-6
	PCB 20/10	**0.9768**	2.9096	17.9897	**1.0075 E-3**	**5.2994 E-6**	20.4366	**0.7545**	0.6426	**2.4470**	2.2790	1.40 E-3	NA	0

* Simulations do not include CDR; Jitter is applied using COM method; Maximum 1M symbols

Summary of the Case Results (1E-3)

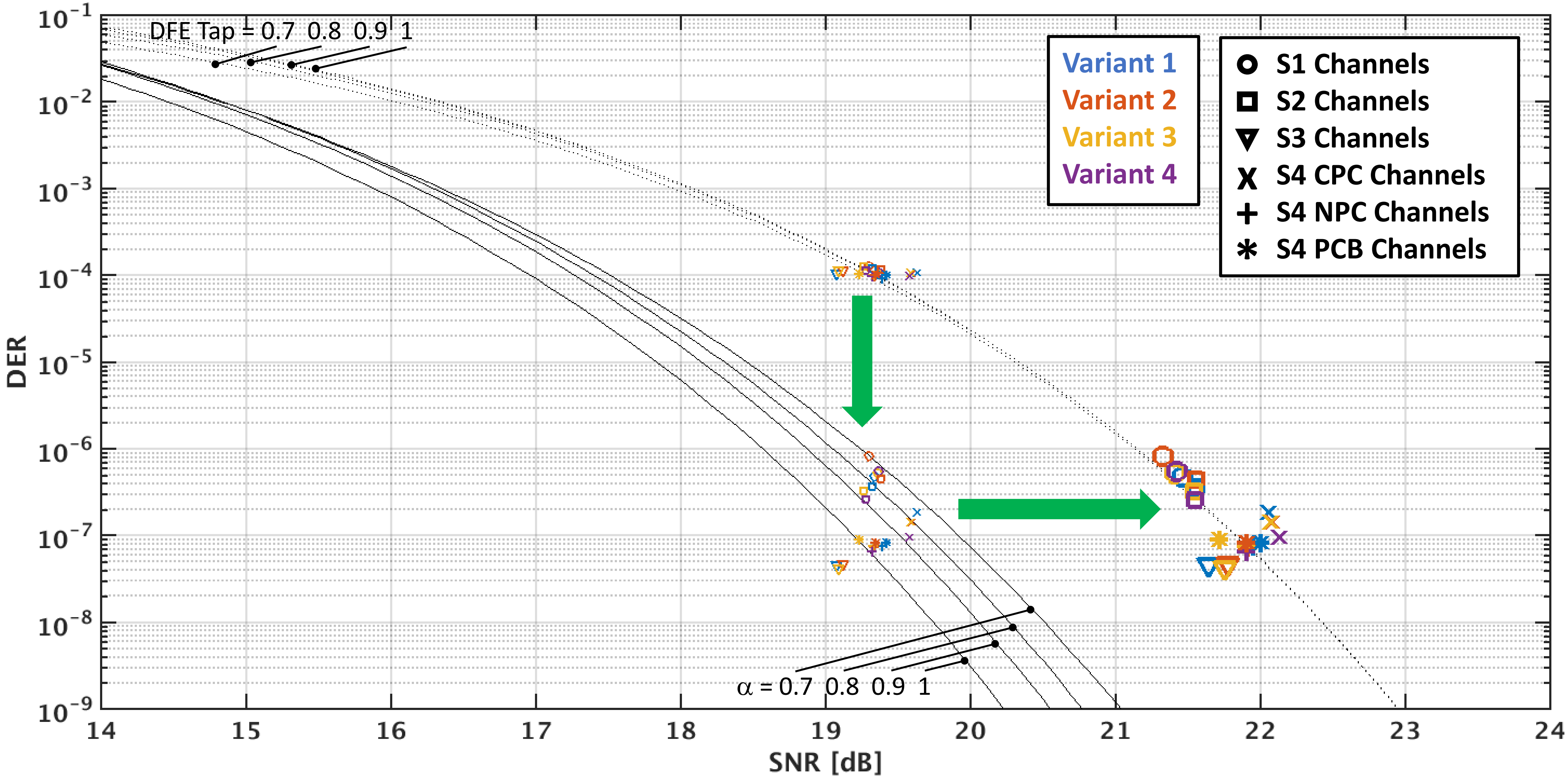


Summary of the Case Results (1E-4)

Channel	Variant	DFE Tap $= \alpha$	Theoretical Coding Gain [dB]	SNR_{DFE} [dB]	DER_{DFE}	DER_{MLSE}	$SNR_{DFE,equivalent}$ [dB]	Nosie Scaling Factor		$\Delta SNR = \Delta COM$ [dB]	DER Ratio [Order of Magnitude]	DER Simulation Results *		
								Total	for Simulation			DFE	MLSE	$DFE_{Equivalent}$
S1	Channel 1	**0.8121**	2.1999	19.3358	**1.1358 E-4**	**4.8160 E-7**	21.4596	**0.7831**	0.7105	**2.1238**	2.3726	6.5 E-5	NA	NA
	Channel 2	**0.7677**	2.0122	19.2972	**1.2762 E-4**	**8.1949 E-7**	21.3249	**0.7918**	0.7413	**2.0277**	2.1924	9.2 E-5	NA	NA
	Channel 3	**0.7746**	2.0413	19.3574	**1.0349 E-4**	**5.1857 E-7**	21.4090	**0.7896**	0.7089	**2.0516**	2.3001	1.24 E-4	NA	NA
	Channel 4	**0.7778**	2.0548	19.3624	**1.0649 E-4**	**5.4976 E-7**	21.4184	**0.7892**	0.7267	**2.0559**	2.2871	1.58 E-4	NA	NA
S2	Case 1	**0.8710**	2.4519	19.3156	**1.2101 E-4**	**3.6782 E-7**	21.5521	**0.7730**	0.7497	**2.2365**	2.5172	2.71 E-4	NA	NA
	Case 2	**0.8382**	2.3112	19.3738	**1.1817 E-4**	**4.4691 E-7**	21.5464	**0.7787**	0.7607	**2.1727**	2.4223	2.23 E-4	NA	NA
	Case 3	**0.8883**	2.5262	19.2597	**1.2679 E-4**	**3.3241 E-7**	21.5347	**0.7696**	0.7341	**2.2751**	2.5814	4.01 E-4	NA	NA
	Case 4	**0.8803**	2.4919	19.2705	**1.1275 E-4**	**2.6353 E-7**	21.5379	**0.7702**	0.7243	**2.2674**	2.6313	2.4 E-4	NA	NA
S3	Conventional	**0.9704**	2.8818	19.0707	**1.0416 E-4**	**4.6160 E-8**	21.6348	**0.7444**	0.4914	**2.5641**	3.3535	1.93 E-4	NA	NA
	CPP	**0.9978**	3.0006	19.1129	**1.1263 E-4**	**4.6876 E-8**	21.7741	**0.7361**	0.4644	**2.6613**	3.3807	9.3 E-5	NA	NA
	NCC	**0.9966**	2.9957	19.0863	**1.1472 E-4**	**4.2448 E-8**	21.7498	**0.7359**	0.3967	**2.6636**	3.4318	1.89 E-4	NA	NA
S4	CPC 30/15	**0.9437**	2.7659	19.6268	**1.0728 E-4**	**1.8526 E-7**	22.0459	**0.7569**	0.7238	**2.4190**	2.7627	1.09 E-4	NA	NA
	CPC 30/20	**0.9618**	2.8445	19.5849	**1.0578 E-4**	**1.4261 E-7**	22.0673	**0.7514**	0.7177	**2.4824**	2.8703	7.5 E-5	NA	NA
	CPC 35/15	**0.9621**	2.8456	19.5808	**1.0668 E-4**	**1.4455 E-7**	22.0637	**0.7514**	0.7174	**2.4829**	2.8680	7.0 E-5	NA	NA
	CPC 35/20	**0.9803**	2.9249	19.5680	**1.0012 E-4**	**9.6782 E-8**	22.1262	**0.7449**	0.7080	**2.5582**	3.0147	1.01 E-4	NA	NA
	NPC 30/15	**0.9775**	2.9125	19.3856	**9.4633 E-5**	**7.5737 E-8**	21.9409	**0.7451**	0.6602	**2.5553**	3.0967	1.19 E-4	NA	NA
	NPC 30/20	**0.9814**	2.9297	19.3345	**1.0004 E-4**	**7.5113 E-8**	21.9067	**0.7437**	0.6453	**2.5722**	3.1245	1.54 E-4	NA	NA
	NPC 35/15	**0.9817**	2.9309	19.3297	**1.0142 E-4**	**7.6746 E-8**	21.9008	**0.7436**	0.6446	**2.5729**	3.1211	1.24 E-4	NA	NA
	NPC 35/20	**0.9816**	2.9306	19.3192	**9.8216 E-5**	**6.6343 E-8**	21.8977	**0.7431**	0.6248	**2.5785**	3.1704	1.94 E-4	NA	NA
	PCB 10/10	**0.9855**	2.9474	19.4163	**1.0046 E-4**	**8.2442 E-8**	21.9985	**0.7428**	0.6760	**2.5823**	3.0859	1.14 E-4	NA	NA
	PCB 15/10	**0.9780**	2.9148	19.3419	**1.0110 E-4**	**8.2962 E-8**	21.8978	**0.7451**	0.6780	**2.5559**	3.0859	1.38 E-4	NA	NA
	PCB 20/10	**0.9555**	2.8170	19.2298	**1.0486 E-4**	**9.2064 E-8**	21.7147	**0.7512**	0.6387	**2.4849**	3.0565	1.77 E-4	NA	NA

* Simulations do not include CDR; Jitter is applied using COM method; Maximum 1M symbols

Summary of the Case Results (1E-4)

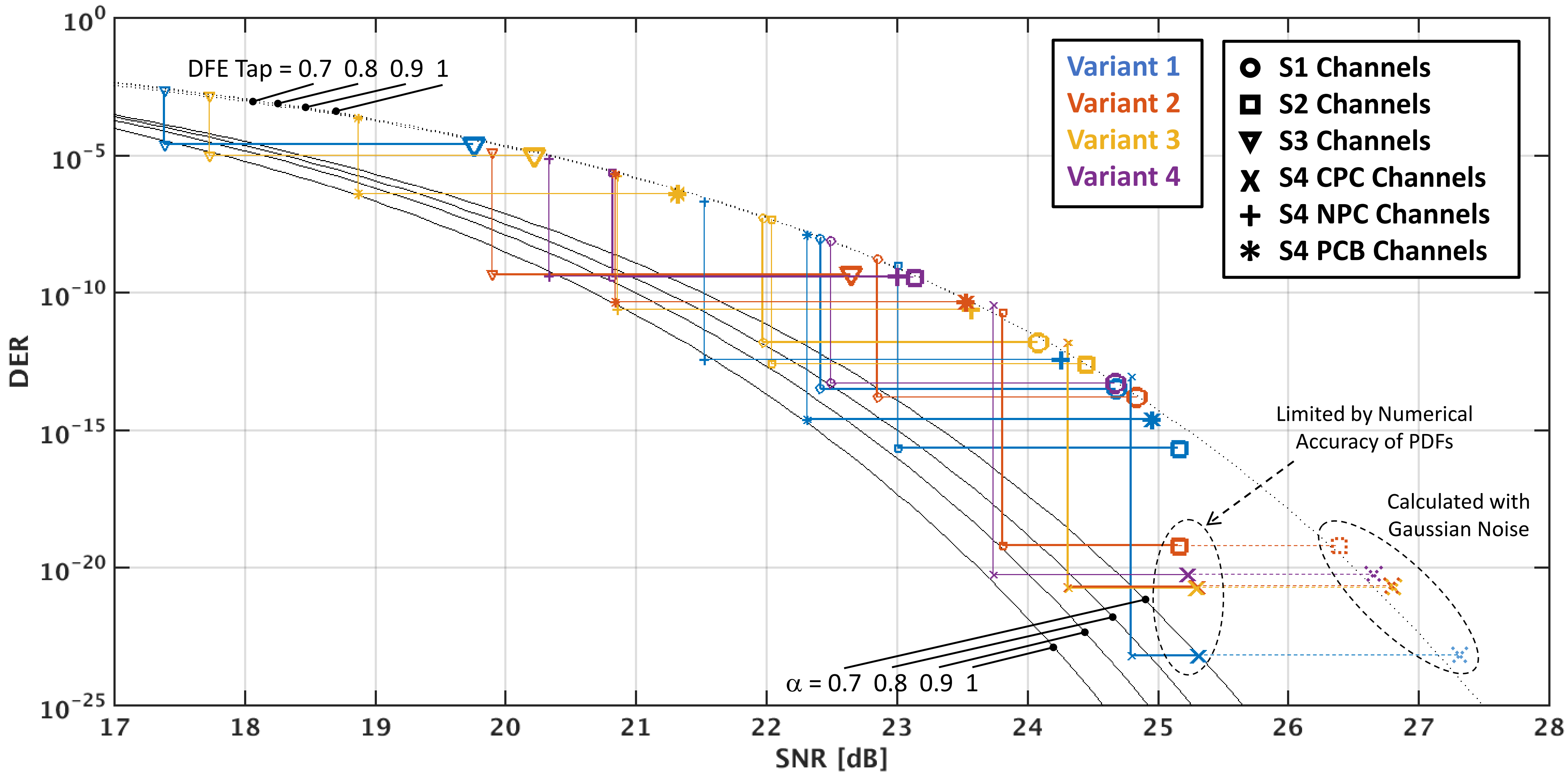


Summary of the Case Results (Native Noise)

Channel	Variant	DFE Tap $= \alpha$	Theoretical Coding Gain [dB]	SNR_{DFE} [dB]	DER_{DFE}	DER_{MLSE}	$SNR_{DFE,equivalent}$ [dB]	Nosie Scaling Factor		$\Delta SNR = \Delta COM$ [dB]	DER Ratio [Order of Magnitude]	DER Simulation Results *		
								Total	for Simulation			DFE	MLSE	$DFE_{Equivalent}$
S1	Channel 1	**0.8116**	2.1977	22.4094	**9.3440 E-9**	**3.2390 E-14**	24.6754	**0.7704**	0.5318	**2.2660**	5.4601	NA	NA	NA
	Channel 2	**0.7272**	1.8437	22.8466	**1.7362 E-9**	**1.6404 E-14**	24.8377	**0.7951**	0.6465	**1.9911**	5.0247	NA	NA	NA
	Channel 3	**0.7655**	2.0029	21.9669	**5.1313 E-8**	**1.6462 E-12**	24.0787	**0.7842**	0.5541	**2.1118**	4.4937	NA	NA	NA
	Channel 4	**0.7850**	2.0849	22.4866	**7.8439 E-9**	**5.1413 E-14**	24.6685	**0.7779**	0.5745	**2.1819**	5.1835	NA	NA	NA
S2	Case 1	**0.8599**	2.4042	23.0054	**1.0158 E-9**	**2.3304 E-16**	25.1529	**0.7810**	0.7324	**2.1475**	6.6394	NA	NA	NA
	Case 2	**0.8893**	2.5308	23.8067	**1.8718 E-11**	**6.6854 E-20**	25.1540	**0.8563**	0.8230	**1.3473**	8.4471	NA	NA	NA
	Case 3	**0.8702**	2.4481	22.0362	**4.7543 E-8**	**2.6394 E-13**	24.4423	**0.7580**	0.7004	**2.4060**	5.2556	NA	NA	NA
	Case 4	**0.8534**	2.3764	20.8167	**2.4914 E-6**	**3.8091 E-10**	23.1275	**0.7664**	0.7153	**2.3108**	3.8156	0	NA	NA
S3	Conventional	**0.9728**	2.8924	17.3785	**2.3461 E-3**	**2.5322 E-5**	19.7544	**0.7607**	0.6036	**2.3759**	1.9668	4.13 E-3	5.1 E-5	1.1 E-5
	CPP	**0.9999**	3.0100	19.8950	**1.3693 E-5**	**4.7054 E-10**	22.6422	**0.7289**	0.2504	**2.7472**	4.4639	1.9 E-5	NA	NA
	NCC	**0.9923**	2.9767	17.7271	**1.5176 E-3**	**9.9057 E-6**	20.2105	**0.7513**	0.5873	**2.4834**	2.1853	1.81 E-3	1.3 E-5	1.2 E-5
S4	CPC 30/15	**0.8389**	2.3141	24.7915	**8.7930 E-14**	**6.7051 E-24**	25.3072	**0.9424**	0.9211	**0.5157**	10.1177	NA	NA	NA
	CPC 30/20	**0.8361**	2.3021	24.3010	**1.6507 E-12**	**2.0676 E-21**	25.2911	**0.8923**	0.8508	**0.9901**	8.9022	NA	NA	NA
	CPC 35/15	**0.8388**	2.3136	24.3061	**1.5934 E-12**	**1.7804 E-21**	25.2872	**0.8932**	0.8519	**0.9812**	8.9518	NA	NA	NA
	CPC 35/20	**0.9843**	2.9419	23.7363	**3.5020 E-11**	**5.8234 E-21**	25.2246	**0.8425**	0.7736	**1.4883**	9.7791	NA	NA	NA
	NPC 30/15	**0.9819**	2.9315	21.5206	**2.2344 E-7**	**3.6820 E-13**	24.2546	**0.7300**	0.5384	**2.7340**	5.7831	NA	NA	NA
	NPC 30/20	**0.9847**	2.9439	20.8551	**1.8751 E-6**	**2.4601 E-11**	23.5614	**0.7323**	0.5527	**2.7063**	4.8821	0	NA	0
	NPC 35/15	**0.9850**	2.9452	20.8528	**1.8886 E-6**	**2.4803 E-11**	23.5602	**0.7322**	0.5523	**2.7073**	4.8816	0	NA	0
	NPC 35/20	**0.9837**	2.9379	20.3274	**7.9269 E-6**	**3.9988 E-10**	23.0001	**0.7351**	0.5559	**2.6727**	4.2972	1.2 E-5	NA	0
	PCB 10/10	**0.9906**	2.9693	22.3082	**1.3449 E-8**	**2.6110 E-15**	24.9509	**0.7377**	0.5560	**2.6427**	6.7119	NA	NA	NA
	PCB 15/10	**0.9815**	2.9300	20.8384	**2.2760 E-6**	**4.6077 E-11**	23.5229	**0.7341**	0.6260	**2.6845**	4.6937	5 E-6	NA	0
	PCB 20/10	**0.9542**	2.8113	18.8703	**2.2232 E-4**	**4.0606 E-7**	21.3151	**0.7547**	0.6565	**2.4448**	2.7384	3.81 E-4	NA	3 E-6

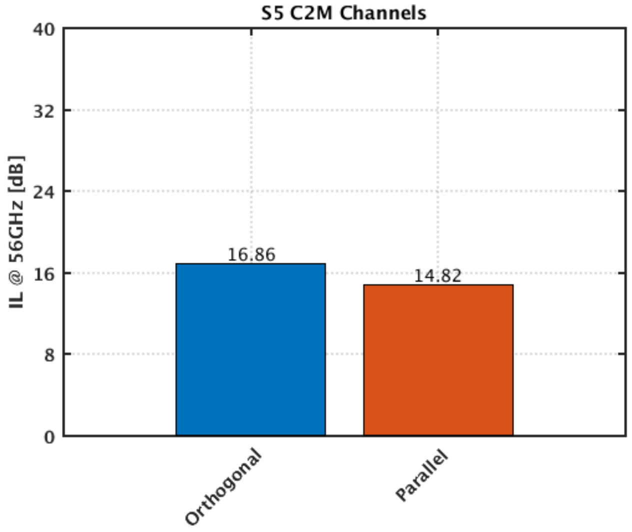
* Simulations do not include CDR; Jitter is applied using COM method; Maximum 1M symbols

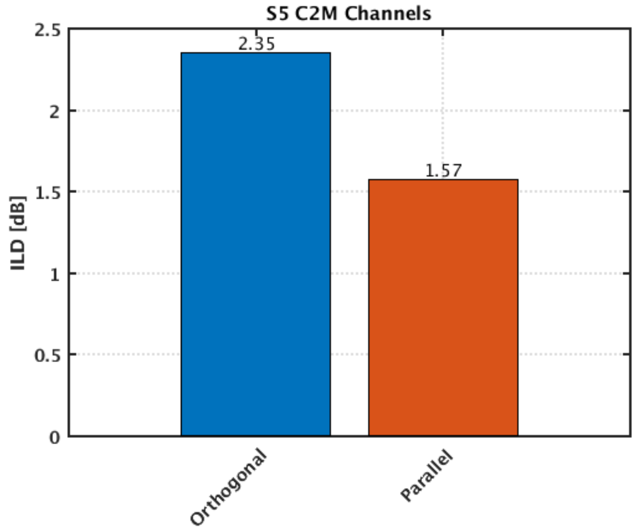
Summary of the Case Results (Native Noise)

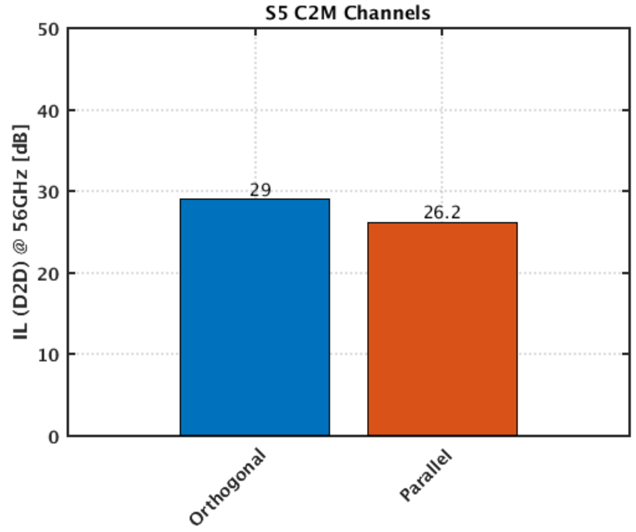


Latest Case Additions (S5)

- rabinovich_3dj_01_230116 (rabinovich_3dj_02_230116 and rabinovich_3dj_03_230116)







Channel	Bit Rate [Gb/s]	Thru Swing [mV]	Fext Swing [mV]	Next Swing [mV]	TX FIR [Pre / Post]	Die C_d [fF] L_s [pH]	C_b [fF]	Package [mm] [Ω]	Rx Filter BW	CTLE Pole/Zero Ratio	DFE [# of Taps]	Rx FFE [Pre / Post]	TX SNR [dB]	Rx Noise [V^2/GHz]	Jitter Rand / DD [UI]	k_N *		
																Native	1E-3	1E-4
S5	224	387	387 x k_N	608 x k_N	3 / 1	40/90/110 130/150/140	30	45 / 0 92	0.75 x f_b	100/2.5/1	1	0 / 8	32.5 - $20\log_{10}(k_N)$	4.1E-9 x k_N^2	0.01 / 0.02 x k_N	1	1.875	1.53
																1	1.925	1.595

* To force more errors to facilitate time-domain simulation verifications

Summary of S5 Case Results

- 1E-3

Channel	Variant	DFE Tap $= \alpha$	Theoretical Coding Gain [dB]	SNR_{DFE} [dB]	DER_{DFE}	DER_{MLSE}	$SNR_{DFE,equivalent}$ [dB]	Nosie Scaling Factor		$\Delta SNR = \Delta COM$ [dB]	DER Ratio [Order of Magnitude]	DER Simulation Results *		
								Total	for Simulation			DFE	MLSE	$DFE_{Equivalent}$
S5	Orthogonal	**0.9516**	2.8001	18.1185	**1.0190 E-3**	**9.3445 E-6**	20.4402	**0.7654**	0.6575	**2.3217**	2.0376	NA	NA	NA
	Parallel	**0.8568**	2.3908	18.1306	**1.0010 E-3**	**1.6731 E-5**	20.2175	**0.7864**	0.7232	**2.0870**	1.7769	NA	NA	NA

* Simulations do not include CDR; Jitter is applied using COM method; Maximum 1M symbols

- 1E-4

Channel	Variant	DFE Tap $= \alpha$	Theoretical Coding Gain [dB]	SNR_{DFE} [dB]	DER_{DFE}	DER_{MLSE}	$SNR_{DFE,equivalent}$ [dB]	Nosie Scaling Factor		$\Delta SNR = \Delta COM$ [dB]	DER Ratio [Order of Magnitude]	DER Simulation Results *		
								Total	for Simulation			DFE	MLSE	$DFE_{Equivalent}$
S5	Orthogonal	**0.9361**	2.7330	19.3735	**1.0109 E-4**	**1.4041 E-7**	21.7834	**0.7577**	0.6073	**2.4100**	2.8573	NA	NA	NA
	Parallel	**0.8623**	2.4142	19.3886	**1.0161 E-4**	**2.7423 E-7**	21.6179	**0.7736**	0.6735	**2.2292**	2.5688	NA	NA	NA

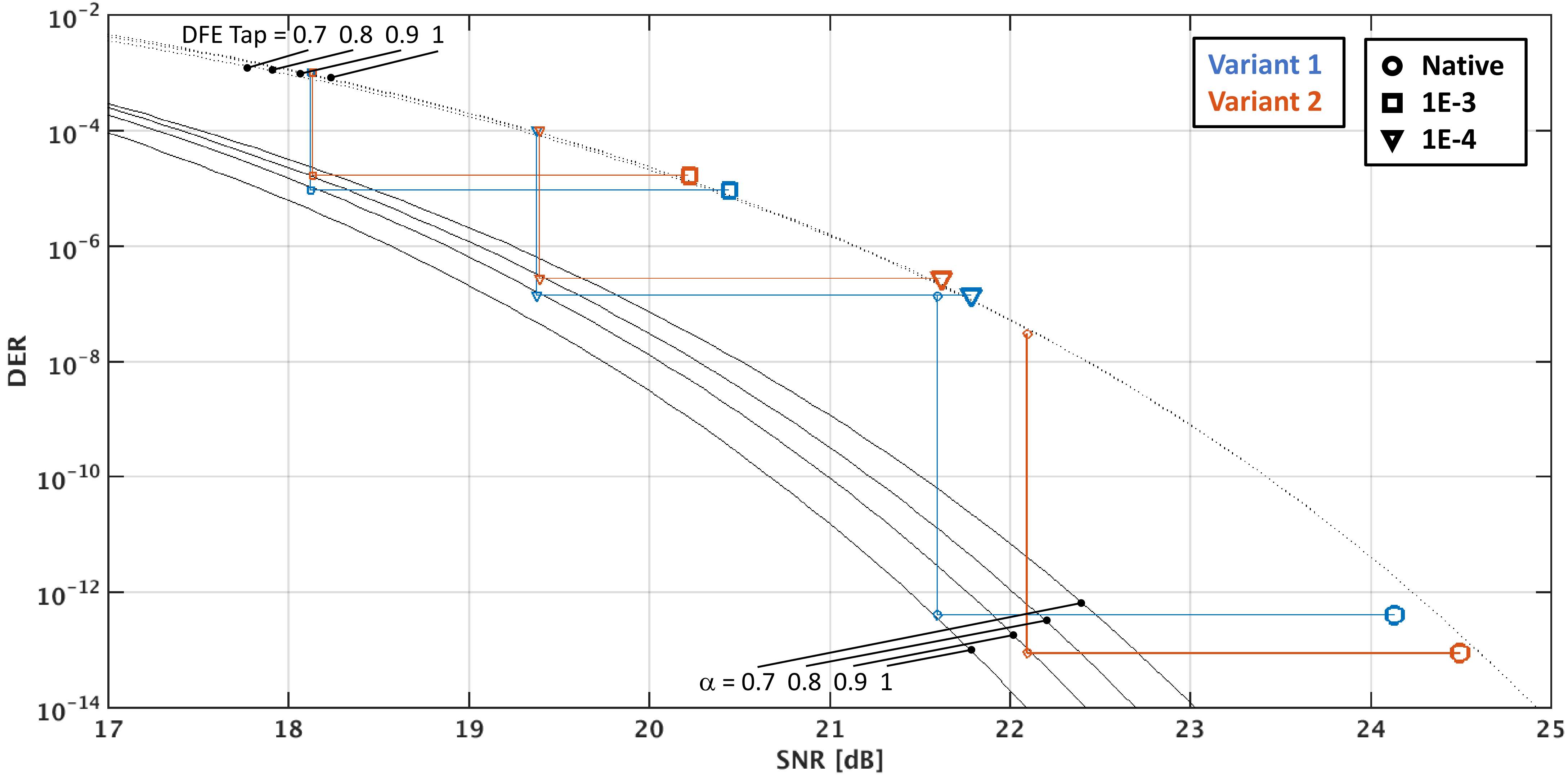
* Simulations do not include CDR; Jitter is applied using COM method; Maximum 1M symbols

- Native

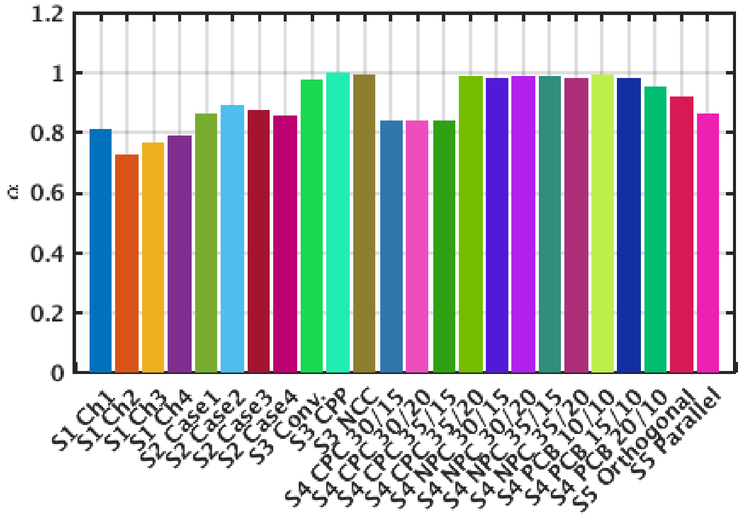
Channel	Variant	DFE Tap $= \alpha$	Theoretical Coding Gain [dB]	SNR_{DFE} [dB]	DER_{DFE}	DER_{MLSE}	$SNR_{DFE,equivalent}$ [dB]	Nosie Scaling Factor		$\Delta SNR = \Delta COM$ [dB]	DER Ratio [Order of Magnitude]	DER Simulation Results *		
								Total	for Simulation			DFE	MLSE	$DFE_{Equivalent}$
S5	Orthogonal	**0.9182**	2.6555	21.5958	**1.3718 E-7**	**4.1091 E-13**	24.1311	**0.7469**	0.3081	**2.5353**	5.5236	NA	NA	NA
	Parallel	**0.8625**	2.4151	22.0927	**3.0338 E-8**	**8.8521 E-14**	24.4931	**0.7585**	0.4872	**2.4004**	5.5349	NA	NA	NA

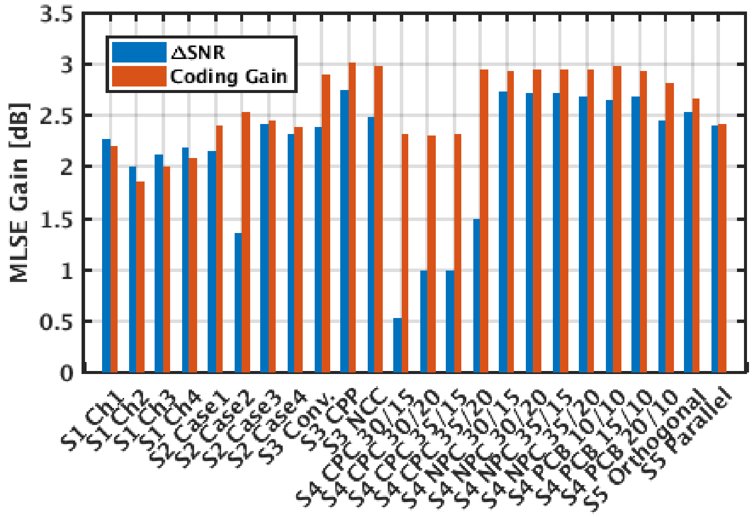
* Simulations do not include CDR; Jitter is applied using COM method; Maximum 1M symbols

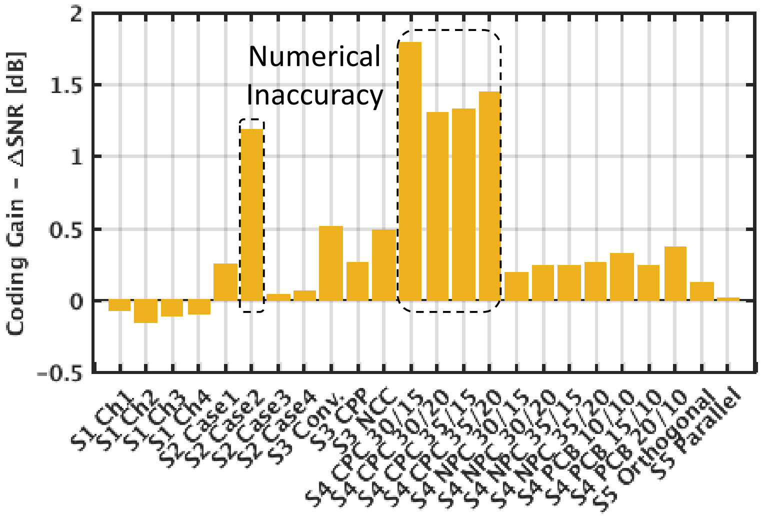
Summary of S5 Case Results



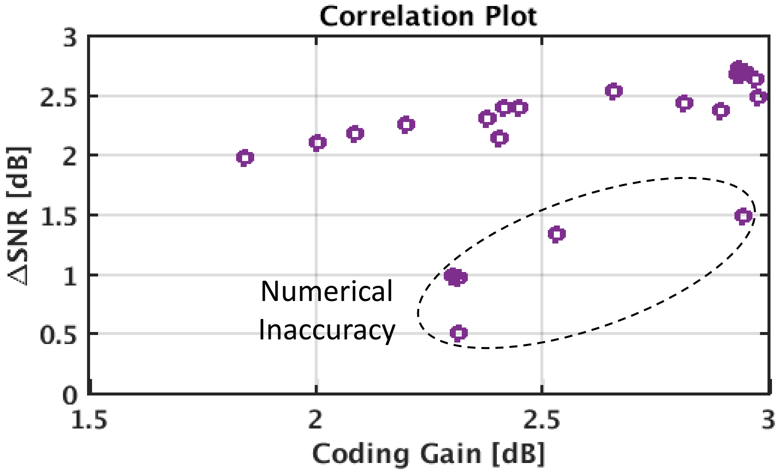
Few Comments on α, ΔSNR, and Coding Gain







- Correlation between coding gain ($10\log_{10}(1+\alpha^2)$) and ΔSNR is not great
- For the lossy channel study cases $0.73 < \alpha < 1$
- From a performance perspective
  - With this high level of post-cursor value, DFE efficiency is poor and wastes 35% to 50% of the useful signal energy
  - In addition, high value of α considerably increases DFE tendency to propagate errors
  - DFE is a much more attractive choice for $\alpha < 0.5$



HUAWEI

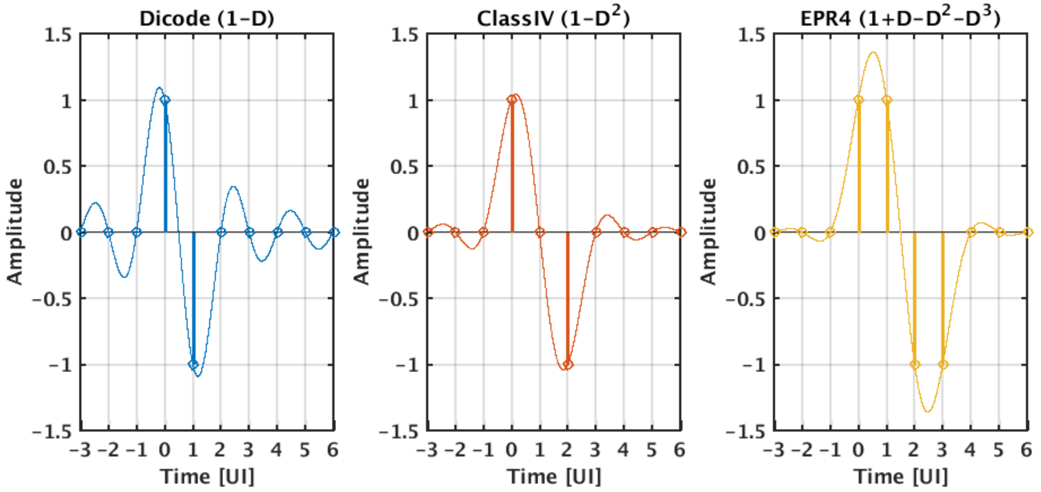
Summary

- This presentation was a recap of the proposal for incorporating performance advantage of MLSE in COM, outlined in “shakiba_3df_01b_2211.pdf” and “shakiba_3dj_01_230116.pdf”
- Validity of the proposal approach and its implementation method were demonstrated by analysis of several channels and cases
- A summary of the DFE and MLSE analysis was provided (Appendix)
- All the equations resulted from analysis of DFE and MLSE were presented
- The proposal boils down to quantifying the equivalent COM advantage of MLSE over DFE
- The equations are COM compatible and directly calculated from COM parameters
- This level of MLSE support is already implemented and available in COM 4.0 Matlab code
- If there is enough interest, equations will continue to be improved and COM Matlab code will be updated
- The proposal is extendable to higher order MLSE as well as other more advanced signal processing techniques

PRML Read Channel, a Success Story

- Operation of writing and reading data on a magnetic media is similar to transmitting and receiving data over a communication channel
- Partial-Response Maximum Likelihood (PRML) is a method developed for high-density packing of stored data on a magnetic media
- High density packing during writing causes ISI between adjacent data during reading
- Handling ISI by using maximum-likelihood sequence estimation (MLSE) enabled tolerating more ISI and led to a substantial and sudden increase in the disk drives in 90s
- Combination of NRZ + PRS (controlled ISI) + MLSE
- First PRML disk drive introduced by IBM in 1990
- Example partial-response polynomials in PRML read channels:

$$(1-D)(1+D)^n \quad \rightarrow \quad \begin{cases} n=0, & 1-D & \mathit{Dicode} \\ n=1, & 1-D^2 & \mathit{ClassIV} \\ n=2, & 1+D-D^2-D^3 & \mathit{EPR4} \end{cases}$$



Appendix A

Calculating SNR for L-PAM

SNR for L-PAM

- Assuming outer PAM levels of ±main and L equi-probable levels

$$PAM\ Level\ Separation = \frac{2main}{L-1}$$

$$PAM\ Levels = -main + \frac{2main}{L-1} l \quad , for\ l = 0, \cdots, L-1$$

$$Signsl\ Power = \frac{main^2}{L} \sum_{l=0}^{L-1} \left(-1 + \frac{l}{L-1}\right)^2 = \frac{1}{3}\frac{L+1}{L-1} main^2$$

(Note that $\frac{1}{L}\sum_{l=0}^{L-1}(PAM\ Levels) = 0$)

$$SNR = \frac{1}{3}\frac{L+1}{L-1}\frac{main^2}{\sigma_{noise}{}^2}$$

- This is the equation used in step 2

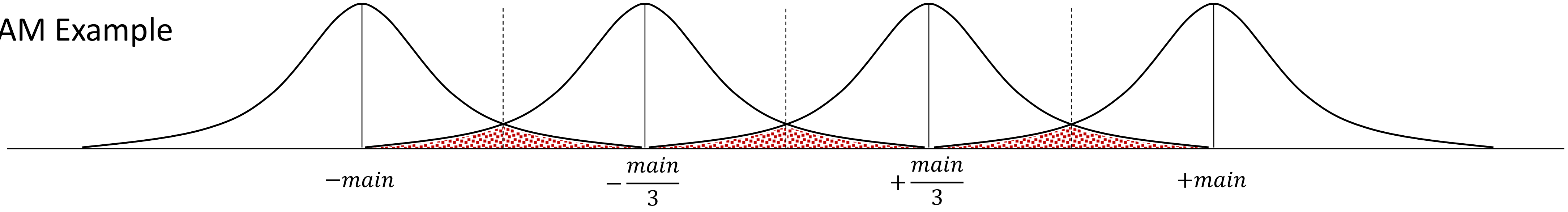
Appendix B

Error analysis of L-PAM 1-Tap DFE

Error Analysis without Error Propagation

- Assuming outer PAM levels of $\pm$main, Gaussian noise, and dominance of adjacent-level errors
- Symbol error probability without error propagation
  - 2L-2 tails extend to wrong decision sides

4-PAM Example



$$DER_{DFE,without\ error\ propagation} \approx \frac{1}{L}(2L-2)Q\left(\frac{main}{(L-1)\sigma_{noise}}\right) = 2\frac{L-1}{L}Q\left(\frac{main}{(L-1)\sigma_{noise}}\right)$$

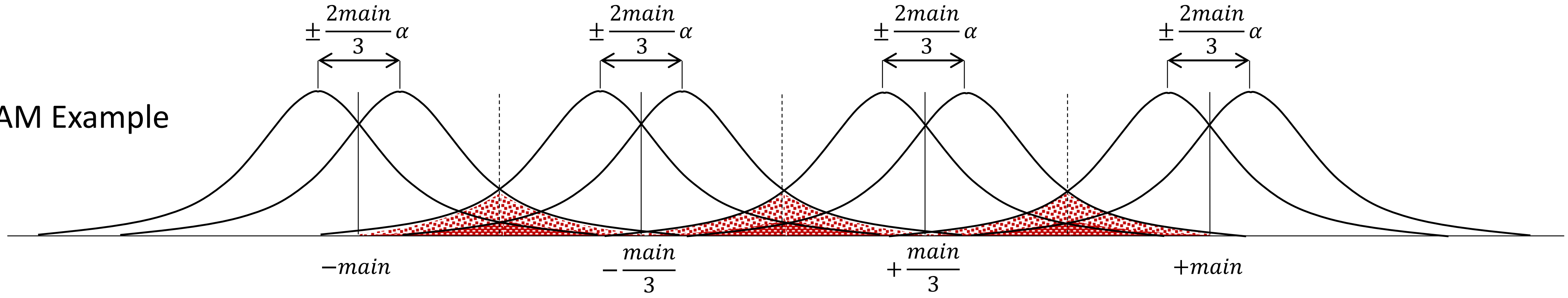
- With error propagation each error symbol extends to a burst of errors
- For the purpose of error calculation, error ratio multiplies by the average burst length

$DER_{DFE} \approx 2\frac{L-1}{L}\overline{BL}_{DFE}Q\left(\frac{main}{(L-1)\sigma_{noise}}\right)$, where assuming exponential distribution for burst lengths $\overline{BL}_{DFE} \approx \frac{1}{1-EPP_{DFE}}$

Error Analysis with Error Propagation

- Error propagation changes each distribution to a bimodal distribution
  - In 2(2L-2) cases error propagation is destructive and tails extend to wrong decision sides

4-PAM Example



for large α (e.g. 0.5 < α < 1)

$$EPP_{DFE} = P(error|previous\ error) \approx \frac{1}{2L}(2L-2)\left(Q\left((1-2\alpha)\frac{main}{(L-1)\sigma_{noise}}\right)+Q\left((1+2\alpha)\frac{main}{(L-1)\sigma_{noise}}\right)\right) \approx \frac{L-1}{L}Q\left((1-2\alpha)\frac{main}{(L-1)\sigma_{noise}}\right)$$

$$DER_{DFE} \approx 2\frac{L-1}{L}\frac{1}{1-EPP_{DFE}}Q\left(\frac{main}{(L-1)\sigma_{noise}}\right) = \frac{2}{\frac{L}{L-1}-Q\left((1-2\alpha)\frac{main}{(L-1)\sigma_{noise}}\right)}Q\left(\frac{main}{(L-1)\sigma_{noise}}\right)$$

Error Analysis with Error Propagation and Arbitrary Noise

- If noise in not Gaussian change the Q function to 1-CDF
- Note that since by definition of Q function its argument is normalized to standard deviation and the argument should now be de-normalized to σ_{noise}

$$DER_{DFE} \approx \frac{2}{\frac{1}{L-1}+CDF_{noise}\left((1-2\alpha)\frac{main}{L-1}\right)}\left(1 - CDF_{noise}\left(\frac{main}{L-1}\right)\right)$$

- The above expression includes the effect of error propagation
- This is the equation used in step 2

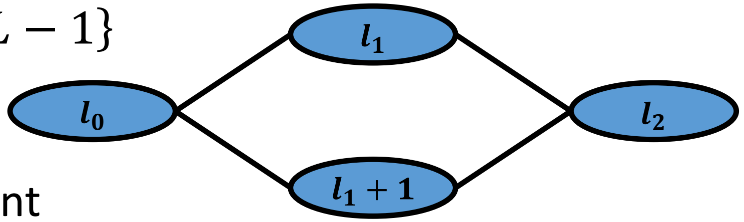
Appendix C

Error analysis of L-PAM 1+αD MLSE

Minimum Distant Error Events

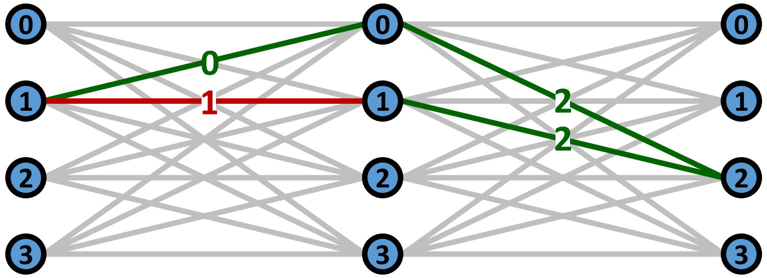
- The obvious one (shortest event)

$l \in \{0, \cdots, L-1\}$



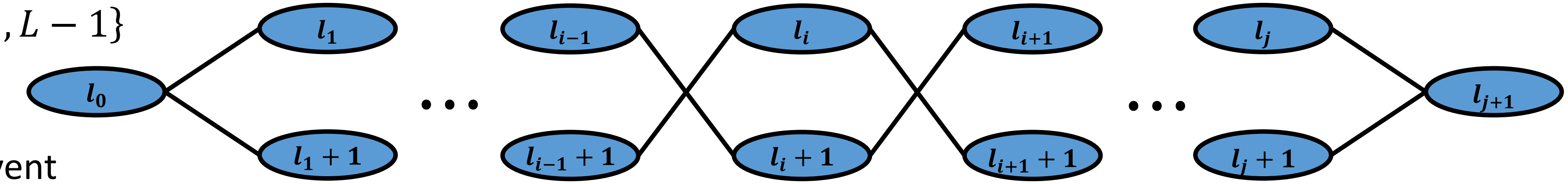
1-Error Event

4-PAM Example
1-Error Event



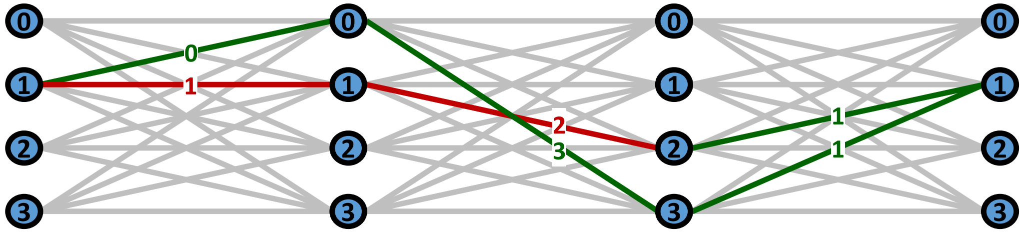
- Not so obvious ones (longer events)

$l \in \{0, \cdots, L-1\}$



j-Error Event

4-PAM Example
2-Error Event



Minimum Distant Error Events

- Assuming outer PAM levels of ±main, the Euclidean distance for the obvious short error event is $2\frac{main}{L-1}\sqrt{1+\alpha^2}$
- The Euclidean distance for the longer error events is $2\frac{main}{L-1}\sqrt{1+(j-1)(1-\alpha)^2+\alpha^2}$ and the burst of errors it entails has a length of j ($j \geq 1$)
- Note that for $j = 1$, this becomes the same as the short error event
- Also note that as α approaches 1 the Euclidean distance of all of the longer error events approaches the distance of the short error event
- This is the error propagation mechanism in the MLSE and is maximized for $\alpha = 1$
- Combinational counting reveals that the fractional frequency of these error events is $2\left(\frac{L-1}{L}\right)^j$

Error Analysis

- Putting together fractional frequency, number of errors, and Euclidean distance for individual events and summation over all the events results in the following overall decision error ratio of the MLSE with Gaussian noise

$$DER_{MLSE} \approx 2\sum_{j=1}^{\infty} j\left(\frac{L-1}{L}\right)^{j} Q\left(\sqrt{1+(j-1)(1-\alpha)^2+\alpha^2}\,\frac{main}{(L-1)\sigma_{noise}}\right)$$

which for arbitrary noise with a known CDF, and after de-normalization, becomes the following expression

$$DER_{MLSE} \approx 2\sum_{j=1}^{\infty} j\left(\frac{L-1}{L}\right)^{j}\left(1-CDF_{noise}\left(\sqrt{1+(j-1)(1-\alpha)^2+\alpha^2}\,\frac{main}{L-1}\right)\right)$$

- This is the equation used in step 3

Analysis of the Conceptual Equivalent DFE

SNR of the ‘Equivalent‘ DFE

- How much does SNR need to increase so that a conceptual equivalent DFE performs as well as the MLSE?

$$DER_{DFE,equivalent} = DER_{MLSE}$$

$$\frac{2}{\frac{L}{L-1} - Q\left((1-2\alpha)\sqrt{\frac{3}{L^2-1}SNR_{DFE,equivalent}}\right)} Q\left(\sqrt{\frac{3}{L^2-1}SNR_{DFE,equivalent}}\right) = DER_{MLSE}$$

- Solving this equation requires iterations, but noticing that the Q function in the denominator of left hand side is a weak function of its argument, particularly $SNR_{DFE,equivalent}$ (which only changes from SNR_{DFE} by as much as a factor of 2), $SNR_{DFE,equivalent}$ can be replaced with SNR_{DFE} to avoid iterations with negligible accuracy penalty, yielding

$$SNR_{DFE,equivalent} = SNR_{DFE}\left(\frac{(L-1)\sigma_{noise}}{main} Q^{-1}\left(\frac{1}{2} DER_{MLSE}\left(\frac{L}{L-1} - Q\left((1-2\alpha)\frac{main}{(L-1)\sigma_{noise}}\right)\right)\right)\right)^2$$

Noise of the ‘Equivalent‘ DFE

- How much does noise need to decrease to give the same increase in SNR so that the conceptual equivalent DFE performs as well as the MLSE?

$$SNR_{DFE} = \frac{1}{3}\frac{L+1}{L-1}\frac{main^2}{\sigma_{noise}{}^2} \qquad SNR_{DFE,equivalent} = \frac{1}{3}\frac{L+1}{L-1}\frac{main^2}{\sigma_{noise,equivalent}{}^2}$$

$$\sigma_{noise,equivalent} = \frac{1}{\frac{L-1}{main}Q^{-1}\left(\frac{1}{2}DER_{MLSE}\left(\frac{L}{L-1}-Q\left((1-2\alpha)\frac{main}{(L-1)\sigma_{noise}}\right)\right)\right)}$$

‘Equivalent‘ SNR and Noise with Arbitrary Noise

- Change the Q function to 1-CDF, de-normalized, and solve

$$\frac{2}{\frac{1}{L-1}+CDF_{noise}\left((1-2\alpha)\sigma_{noise}\sqrt{\frac{3}{L^2-1}SNR_{DFE,equivalent}}\right)}\left(1-CDF\left(\sigma_{noise}\sqrt{\frac{3}{L^2-1}SNR_{DFE,equivalent}}\right)\right)=DER_{MLSE}$$

- Similarly, replace $\text{SNR}_{\text{DFE,equivalent}}$ in the denominator with SNR_{DFE} to avoid iterations to yield

$$SNR_{DFE,equivalent}=\left(\frac{L-1}{main}CDF_{noise}^{-1}\left(1-\frac{1}{2}DER_{MLSE}\left(\frac{1}{L-1}+CDF_{noise}\left((1-2\alpha)\frac{main}{L-1}\right)\right)\right)\right)^2 SNR_{DFE}$$

- Equivalently, this increase in SNR can be expressed as a decrease in noise

$$\sigma_{noise,equivalent}=\frac{1}{\frac{L-1}{main}CDF_{noise}^{-1}\left(1-\frac{1}{2}DER_{MLSE}\left(\frac{1}{L-1}+CDF_{noise}\left((1-2\alpha)\frac{main}{L-1}\right)\right)\right)}\sigma_{noise}$$

- Last two equations are the equations used in step 4