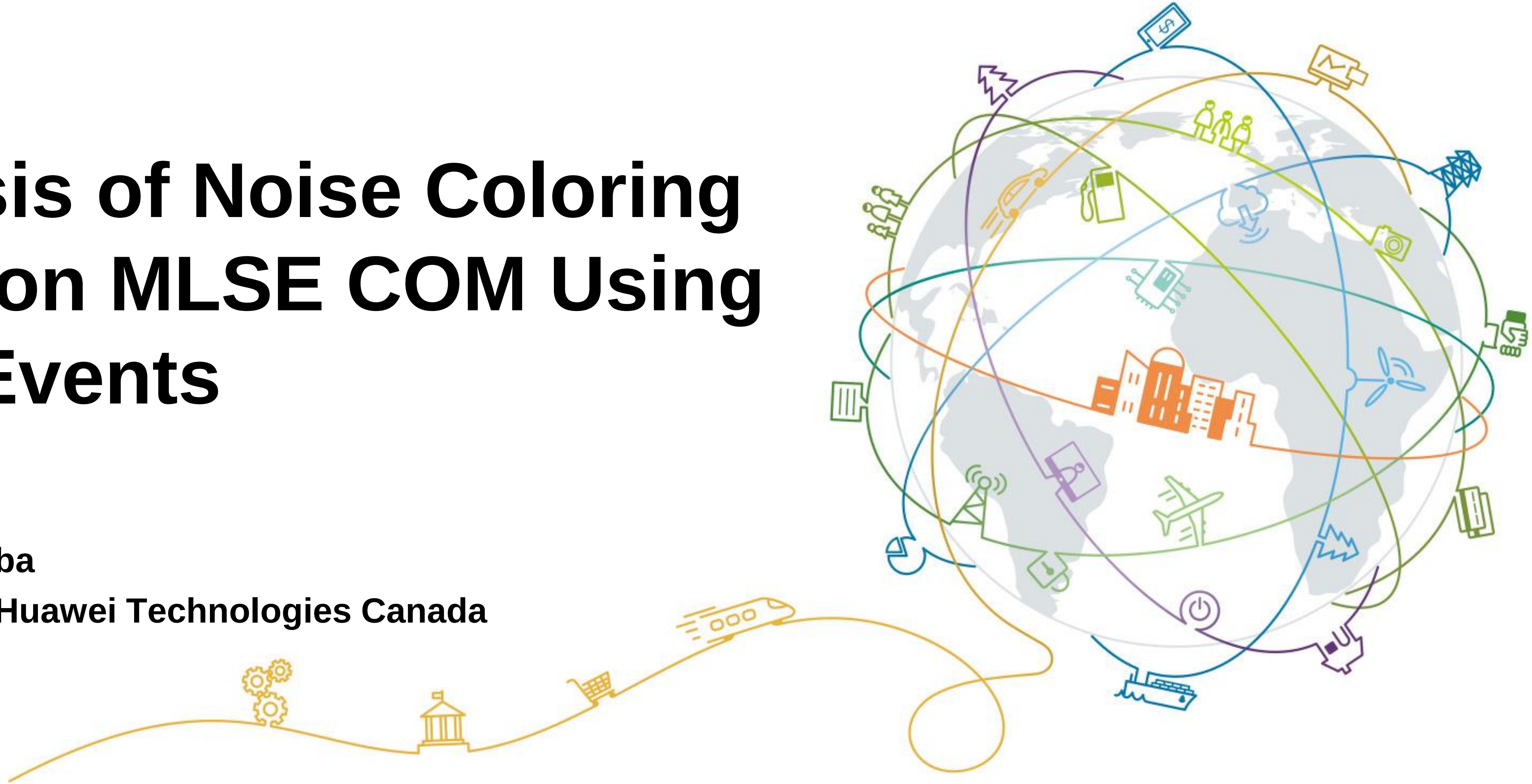
4/18/2023

Analysis of Noise Coloring Effect on MLSE COM Using Error Events



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Outline

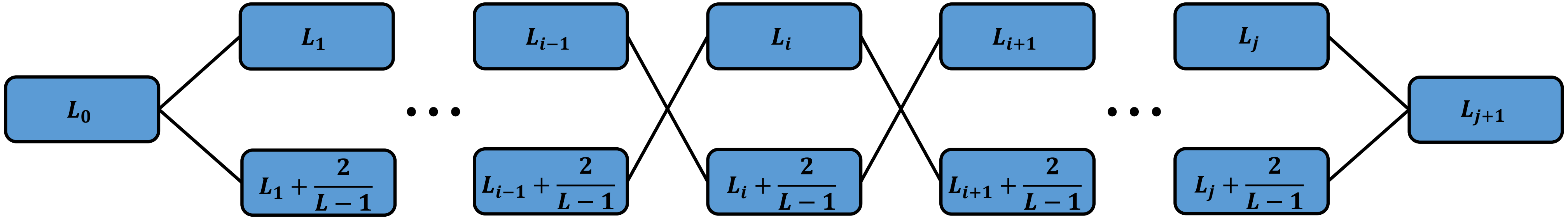
- Introduction and Background
- Analysis of Minimum-Distance Error Event
  - Start of the Error / Propagation of the Error / End of the Error
- MLSE Sequence Noise
  - Correlation Matrix / Power
- Conceptual Examples
- Probability of MLSE Error Event (DER_{MLSE})
- Few Scenarios
- **Suggestions to Update the MLSE COM Equations and Changes to the Steps (Slides 17-19, Main Takeaway)**
- 24 Study Cases
  - Link Parameter / Summary of Results / Noise PSD / Noise Correlation / MLSE COM (ΔSNR) and DER Comparison
- Summary and Conclusions
- Appendix
  - A. Error Event Rate of L-PAM 1-TAP DFE
  - B. Analysis of the Conceptual Equivalent DFE

Introduction

- Contributions shakiba_3df_01b_2211, shakiba_3dj_01_230116, and shakiba_3dj_elec_01_230223 that introduced and explained a 5-step method for calculating MLSE COM (ΔCOM) assumed white (non-Gaussian) noise
- This contribution is a summary of the effect of noise coloring MLSE COM
- The analysis is also applicable to cases where input noises are pre-colored (in this case COM needs to be modified to import the input noise PSDs)
- In addition, the previous contributions were based on PAM symbol error rate as the measure of performance (in the last two contributions "DER" was used to denote symbol error rate)
- This contribution considers the error event rate (or detector error ratio) (hereon "DER" denotes error event rate for consistency with COM terminology)
- To increase the accuracy of calculations, a more comprehensive method for calculating the PDF of the MLSE noise is suggested

Background

- Contribution shakiba_3df_01b_2211.pdf showed that error events of an L-PAM $1+\alpha D$ MLSE are dominated by a zig-zag pattern in the form of alteranting adjacent levels:

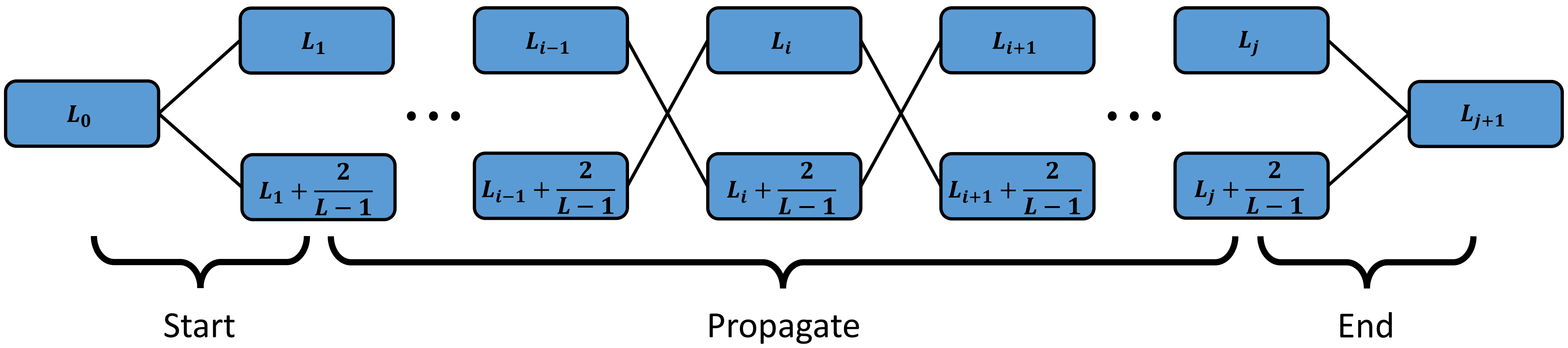


$$L_i \in \begin{cases} -1,-1+\frac{2}{L-1},\cdots,+1-\frac{2}{L-1},+1 & i=0,j+1 \\ -1,-1+\frac{2}{L-1},\cdots,+1-\frac{2}{L-1} & i=1,\dots,j \end{cases}$$

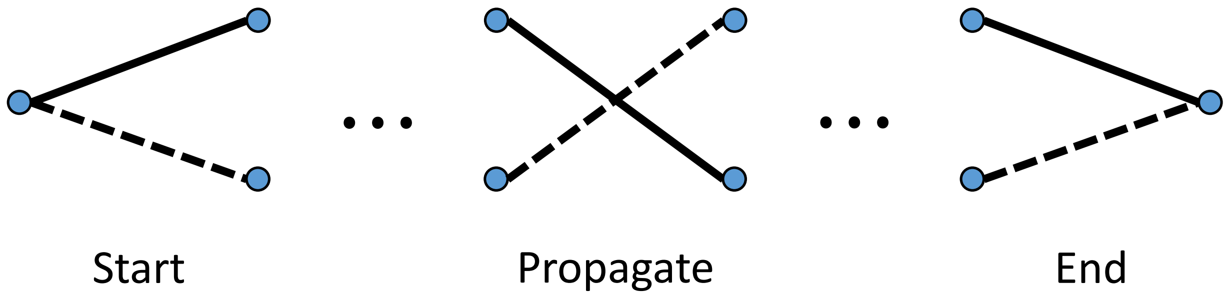
- Rather than assuming uncorrelated noise samples, which simplified calculation of the MLSE sequence noise, to include the effect of coloring the sequence noise must be calculated with the correlation between samples taken into consideration

Analysis of Minimum-Distance Error Event

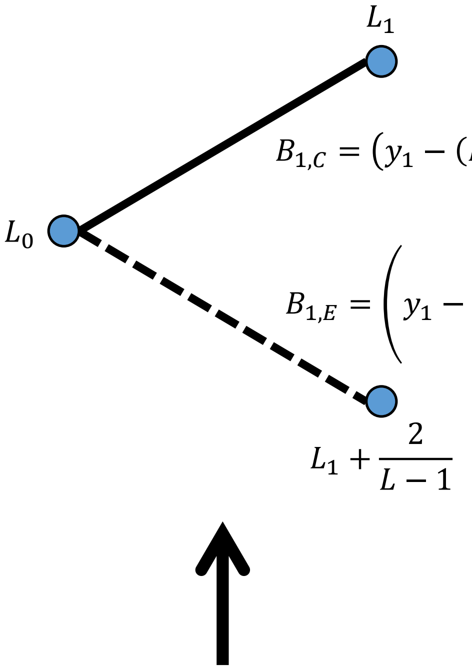
- The alternating error event has three distinct sections:



- Corresponding to these sections, we analyze three following trellis transitions and without loss of generality consider the solid trace as the correct one:



Start of the Error



$$B_{1,C} = \left(y_1 - (L_1 + \alpha L_0)\right)^2 = {n_1}^2$$

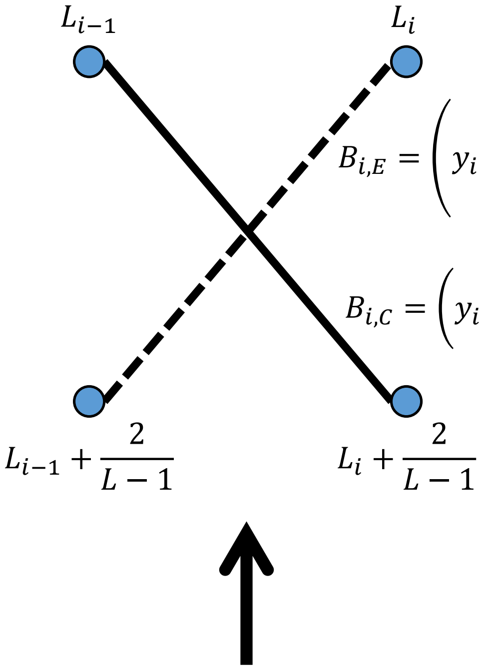
$$B_{1,E} = \left(y_1 - \left(\left(L_1 + \frac{2}{L-1}\right) + \alpha L_0\right)\right)^2 = \left(n_1 - \frac{2}{L-1}\right)^2$$

Time Step 1

$$y_1 = x_1 + n_1$$
$$x_1 = L_1 + \alpha L_0$$

Propagation of the Error

Even i ($i = 2, \cdots, j$)



$$B_{i,E} = \left(y_i - \left(L_i + \alpha\left(L_{i-1} + \frac{2}{L-1}\right)\right)\right)^2 = \left(n_i + \frac{2}{L-1}(1-\alpha)\right)^2$$

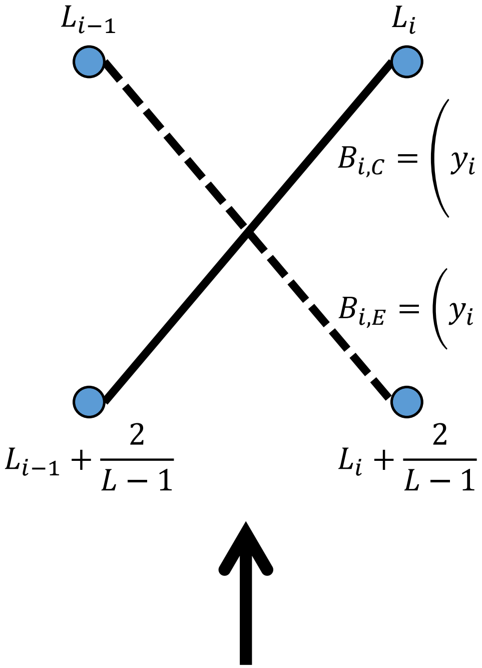
$$B_{i,C} = \left(y_i - \left(L_i + \frac{2}{L-1}\right) + \alpha L_{i-1}\right)^2 = {n_i}^2$$

Time Step i

$$y_i = x_i + n_i$$

$$x_i = \left(L_i + \frac{2}{L-1}\right) + \alpha L_{i-1}$$

Odd i ($i = 2, \cdots, j$)



$$B_{i,C} = \left(y_i - \left(L_i + \alpha\left(L_{i-1} + \frac{2}{L-1}\right)\right)\right)^2 = {n_i}^2$$

$$B_{i,E} = \left(y_i - \left(L_i + \frac{2}{L-1}\right) + \alpha L_{i-1}\right)^2 = \left(n_i - \frac{2}{L-1}(1-\alpha)\right)^2$$

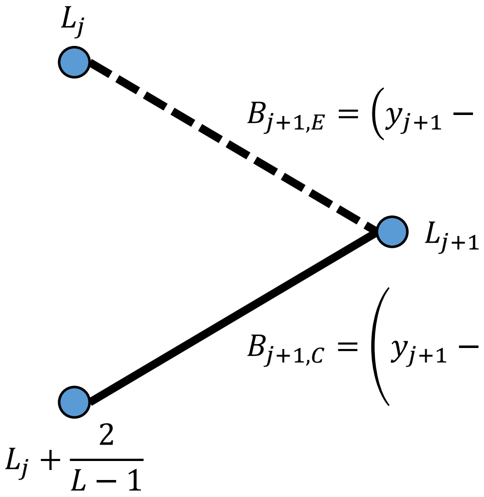
Time Step i

$$y_i = x_i + n_i$$

$$x_i = L_i + \alpha\left(L_{i-1} + \frac{2}{L-1}\right)$$

End of the Error

Odd $j + 1$ (Even j)



$$B_{j+1,E} = \left(y_{j+1} - (L_{j+1} + \alpha L_j)\right)^2 = \left(n_{j+1} + \frac{2}{L-1}\alpha\right)^2$$

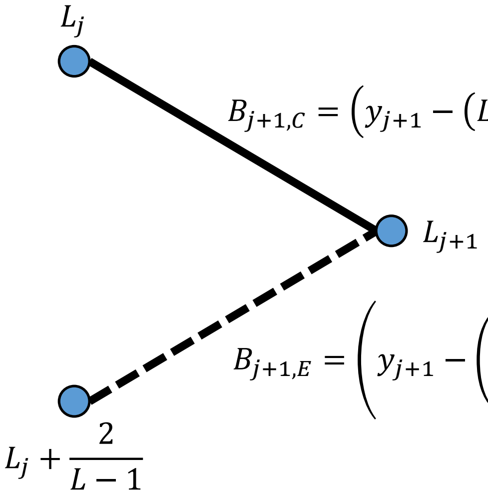
$$B_{j+1,C} = \left(y_{j+1} - \left(L_{j+1} + \alpha\left(L_j + \frac{2}{L-1}\right)\right)\right)^2 = {n_{j+1}}^2$$

Time Step $j + 1$

$$y_{j+1} = x_{j+1} + n_{j+1}$$

$$x_{j+1} = L_{j+1} + \alpha\left(L_j + \frac{2}{L-1}\right)$$

Even $j + 1$ (Odd j)



$$B_{j+1,C} = \left(y_{j+1} - (L_{j+1} + \alpha L_j)\right)^2 = {n_{j+1}}^2$$

$$B_{j+1,E} = \left(y_{j+1} - \left(L_{j+1} + \alpha\left(L_j + \frac{2}{L-1}\right)\right)\right)^2 = \left(n_{j+1} - \frac{2}{L-1}\alpha\right)^2$$

Time Step $j + 1$

$$y_{j+1} = x_{j+1} + n_{j+1}$$

$$x_{j+1} = L_{j+1} + \alpha L_j$$

MLSE Sequence Noise

- MLSE makes a wrong decision if the accumulated MSE of the error path is less than that of the correct path:

$$\sum_{i=1}^{j+1} B_{i,E} < \sum_{i=1}^{j+1} B_{i,C} \quad \rightarrow \quad n_1 - (1-\alpha)\sum_{i=2}^{j}(-1)^i n_i + \alpha(-1)^{j+1} n_{j+1} > \frac{1}{L-1}(1+(j-1)(1-\alpha)^2+\alpha^2)$$

- The sequence noise of MLSE (LHS) is a linear combination of $j+1$ samples of noise :

$$n_{jEE} = n_1 - (1-\alpha)\sum_{i=2}^{j}(-1)^i n_i + \alpha(-1)^{j+1} n_{j+1}$$

- These are samples of the same random noise taken at consecutive $(j+1)$ T-spaced intervals
- In case of white noise, noise samples (n_i) are identically distributed and independent random variables and will be weighted and power summed:

$$\sigma_{noise,jEE}{}^2 = \sigma_{noise}{}^2(1+(j-1)(1-\alpha)^2+\alpha^2) \quad , White$$

- However, if noise is colored the samples are identically distributed but dependent random variables and correlation between samples must be known to calculate the sequence noise

Correlation Matrix of the Sequence Noise

- Assuming noise is stationary, its autocorrelation function is inverse Fourier transform of its PSD:

$$R_{NN}(\tau) = F^{-1}\{PSD_{noise}\}$$

- Correlation coefficients are $(2j+1)$ T-spaced (UI) samples $(\rho_{-j},\cdots,\rho_0,\cdots,\rho_j)$ of the normalized autocorrelation function:

$$\rho_i = \frac{R_{NN}(iT)}{R_{NN}(0)={\sigma_{noise}}^2} \quad i=-j,\cdots,0,\cdots,j$$

- Correlation matrix of the sequence noise can be calculated from the correlation coefficients and the weights in the linear combination expression:

$$Cov_{noise,jEE} = {\sigma_{noise}}^2 \underbrace{\begin{bmatrix} 1 & -(1-\alpha)\rho_1 & +(1-\alpha)\rho_2 & \cdots & (-1)^{j+1}\alpha\rho_j \\ -(1-\alpha)\rho_{-1} & (1-\alpha)^2 & -(1-\alpha)^2\rho_1 & \cdots & (-1)^j\alpha(1-\alpha)\rho_{j-1} \\ +(1-\alpha)\rho_{-2} & -(1-\alpha)^2\rho_{-1} & (1-\alpha)^2 & \cdots & (-1)^{j-1}\alpha(1-\alpha)\rho_{j-2} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ (-1)^{j+1}\alpha\rho_{-j} & (-1)^j\alpha(1-\alpha)\rho_{-(j-1)} & (-1)^{j-1}\alpha(1-\alpha)\rho_{-(j-2)} & \cdots & \alpha^2 \end{bmatrix}}_{\text{Correlation Matrix, } \rho_{noise,jEE}}$$

Power of the MLSE Sequence Noise

- Variance (in this case power) of the sequence noise is sum of all the elements of the covariance matrix:

$$\sigma_{noise,jEE}{}^2 = \sum_{vertical}\sum_{horizontal}\left(Cov_{noise,jEE}\right) = \sigma_{noise}{}^2\sum_{vertical}\sum_{horizontal}\left(\rho_{noise,jEE}\right)$$

$$\sigma_{noise,jEE}{}^2 = \sigma_{noise}{}^2(1 + (j-1)(1-\alpha)^2 + \alpha^2 + Sum\ of\ Red\ Terms)$$

- The term in red is the increment in the sequence noise power due to coloring
- Depending on the polarities of the correlation coefficients this increment could be positive or negative

Conceptual Examples

- Two HPF and LPF examples are chosen to showcase extreme noise coloring scenarios by either amplifying or attenuating the high-frequency energy
- In reality, noise components experience different shaping

Correlation Matrix, $\rho_{noise,5EE}$

Correlation Matrix for $\alpha = 0.8$

$$White: \begin{bmatrix} 1.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & -0.0000 \\ 0.0000 & 0.0400 & 0.0000 & 0.0000 & 0.0000 & -0.0000 \\ 0.0000 & 0.0000 & 0.0400 & 0.0000 & 0.0000 & -0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0400 & 0.0000 & -0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0400 & -0.0000 \\ -0.0000 & -0.0000 & -0.0000 & -0.0000 & -0.0000 & 0.6400 \end{bmatrix}$$

$$LPF: \begin{bmatrix} 1.0000 & -0.0702 & -0.0266 & 0.0014 & 0.0059 & -0.0067 \\ -0.0702 & 0.0400 & -0.0140 & -0.0053 & 0.0003 & -0.0047 \\ -0.0266 & -0.0140 & 0.0400 & -0.0140 & -0.0053 & -0.0011 \\ 0.0014 & -0.0053 & -0.0140 & 0.0400 & -0.0140 & 0.0213 \\ 0.0059 & 0.0003 & -0.0053 & -0.0140 & 0.0400 & 0.0562 \\ -0.0067 & -0.0047 & -0.0011 & 0.0213 & 0.0562 & 0.6400 \end{bmatrix}$$

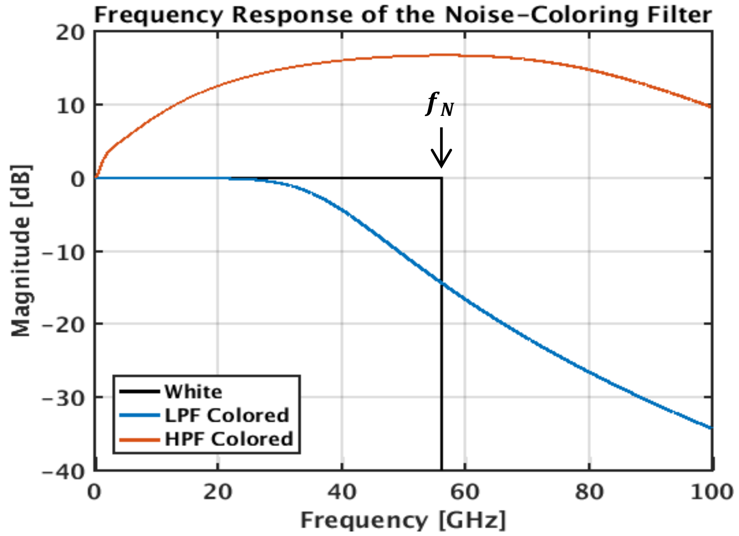
$$HPF: \begin{bmatrix} 1.0000 & 0.0913 & -0.0057 & 0.0021 & -0.0009 & 0.0033 \\ 0.0912 & 0.0400 & 0.0183 & -0.0011 & 0.0004 & 0.0007 \\ -0.0055 & 0.0182 & 0.0400 & 0.0183 & -0.0011 & -0.0017 \\ 0.0019 & -0.0011 & 0.0182 & 0.0400 & 0.0183 & 0.0045 \\ -0.0008 & 0.0004 & -0.0011 & 0.0182 & 0.0400 & 0.0730 \\ 0.0031 & 0.0006 & -0.0016 & 0.0044 & -0.0729 & 0.6400 \end{bmatrix}$$

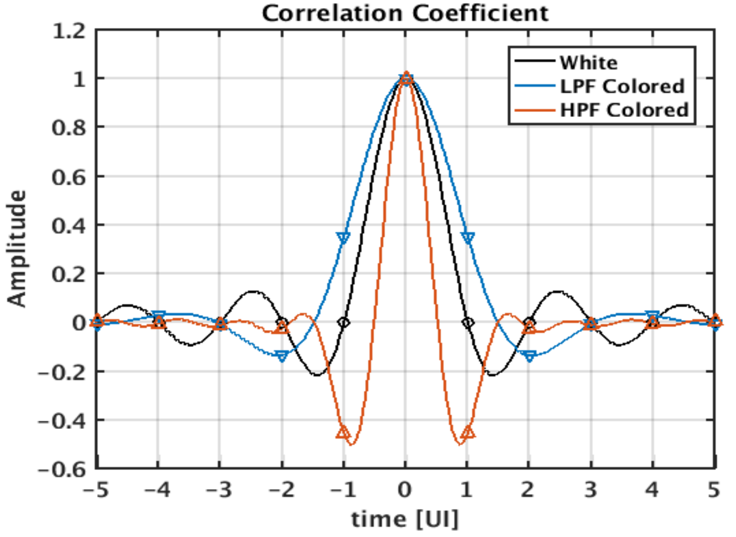
Correlation Matrix for $\alpha = 1$

$$White: \begin{bmatrix} 1.0000 & 0 & 0 & 0 & 0 & -0.0000 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ -0.0000 & 0 & 0 & 0 & 0 & 1.0000 \end{bmatrix}$$

$$LPF: \begin{bmatrix} 1.0000 & 0 & 0 & 0 & 0 & -0.0084 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ -0.0084 & 0 & 0 & 0 & 0 & 1.0000 \end{bmatrix}$$

$$HPF: \begin{bmatrix} 1.0000 & 0 & 0 & 0 & 0 & 0.0041 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0.0039 & 0 & 0 & 0 & 0 & 1.0000 \end{bmatrix}$$





Probability of MLSE Error Event (DER_{MLSE})

- An error event of length j (burst) occurs if:

$$n_{jEE} = n_1 - (1-\alpha)\sum_{i=2}^{j}(-1)^i n_i + \alpha(-1)^{j+1} n_{j+1} > \frac{1}{L-1}\left(1 + (j-1)(1-\alpha)^2 + \alpha^2\right)$$

- Assuming amplitude of $main$ for the L-PAM signal, probability of the j-error event is:

$$P(jEE) = P\left(n_{jEE} > \frac{main}{L-1}\left(1 + (j-1)(1-\alpha)^2 + \alpha^2\right) \quad \& \quad \underset{for\ i=1,2,\cdots}{n_{(j+i)EE}} < \frac{main}{L-1}\left(1 + (j+i-1)(1-\alpha)^2 + \alpha^2\right)\right)$$

- The above expression is very involved and contains infinite series of infinite integrals that cannot be evaluated in a closed-form format, and numerical calculation is intense
- An upper bound (tighter as SNR increases) to this expression can be obtained by dropping the condition on the longer error events:

$$P(jEE) \le P\left(n_{jEE} > \frac{main}{L-1}\left(1 + (j-1)(1-\alpha)^2 + \alpha^2\right)\right) = \begin{cases} 1 - CDF_{noise,jEE}\left(\frac{main}{L-1}\left(1 + (j-1)(1-\alpha)^2 + \alpha^2\right)\right) \\ Q\left(\frac{main}{L-1}\frac{1+(j-1)(1-\alpha)^2+\alpha^2}{\sigma_{noise,jEE}}\right) \quad , Gaussian \end{cases}$$

where $CDF_{noise,jEE}$ is the CDF of the sequence noise of the j-error event

Probability of MLSE Error Event (DER_{MLSE})

- Contribution of each error event to the overall probability of error event (DER) of MLSE is determined by the probability of the event and its fractional frequency:

$$DER_{MLSE} \approx \begin{cases} 2\sum_{j=1}^{\infty}\left(\frac{L-1}{L}\right)^{j}\left(1 - CDF_{noise,jEE}\left(\frac{main}{L-1}\left(1+(j-1)(1-\alpha)^2+\alpha^2\right)\right)\right) \\ 2\sum_{j=1}^{\infty}\left(\frac{L-1}{L}\right)^{j} Q\left(\frac{main}{L-1}\frac{1+(j-1)(1-\alpha)^2+\alpha^2}{\sigma_{noise,jEE}}\right) \quad ,Gaussian \end{cases}$$

- The challenge is that for non-Gaussian noise, $CDF_{noise,jEE}$ (or $PDF_{noise,jEE}$) should be individually calculated for each sequence length
- In the case of white noise ${\sigma_{noise,jEE}}^2 = {\sigma_{noise}}^2(1+(j-1)(1-\alpha)^2+\alpha^2)$, and the above expression can be approximated as:

$$DER_{MLSE} \approx \begin{cases} 2\sum_{j=1}^{\infty}\left(\frac{L-1}{L}\right)^{j}\left(1 - CDF_{noise}\left(\frac{main}{L-1}\sqrt{1+(j-1)(1-\alpha)^2+\alpha^2}\right)\right) \quad ,White\ (See\ the\ note\ on\ the\ next\ slide) \\ 2\sum_{j=1}^{\infty}\left(\frac{L-1}{L}\right)^{j} Q\left(\frac{main}{L-1}\frac{\sqrt{1+(j-1)(1-\alpha)^2+\alpha^2}}{\sigma_{noise}}\right) \quad ,White\ Gaussian \end{cases}$$

which is consistent with the symbol error rate results presented in shakiba_3df_01b_2211.pdf

Few Scenarios

1) White Gaussian noise

   n_{jEE} is also Gaussian with ${\sigma_{noise,jEE}}^2 = {\sigma_{noise}}^2(1 + (j-1)(1-\alpha)^2 + \alpha^2)$

2) Colored Gaussian noise

   n_{jEE} is also Gaussian with ${\sigma_{noise,jEE}}^2 = {\sigma_{noise}}^2(1 + (j-1)(1-\alpha)^2 + \alpha^2 + \textit{Red Terms}\ (\textit{Slide}\ 11)\ )$

3) White non-Gaussian noise

   n_{jEE} has a PDF equal to the convolution of $j+1$ scaled noise PDFs:

$$PDF_{noise,jEE}(x) = PDF_{noise}(x) * \mathrm{conv}_{i=2}^{j} PDF_{noise}\left(-(1-\alpha)(-1)^i x\right) * PDF_{noise}\left(\alpha(-1)^{j+1} x\right)$$

which almost always simplifies to (PDF is usually symmetric):

$$PDF_{noise,jEE}(x) = PDF_{noise}(x) * \mathrm{conv}_{i=2}^{j} PDF_{noise}((1-\alpha)x) * PDF_{noise}(\alpha x)$$

Note that, in the previous contribution further simplification was made by using PDF_{noise} instead of $PDF_{noise,jEE}$ and scaling its argument back by the ratio between σ_{noise} and $\sigma_{noise,jEE}$ to obtain $PDF_{noise,jEE}$ for individual sequence noises

Few Scenarios

4) Colored non-Gaussian noise

There is no closed-form formula for $PDF_{noise,jEE}$, but still

$\sigma_{noise,jEE}{}^2 = \sigma_{noise}{}^2(1 + (j-1)(1-\alpha)^2 + \alpha^2 + Red\ Terms\ (Slide\ 11))$

Numerical calculation could be practically prohibitive as it requires calculating many conditional PDFs for many error lengths

What do we do? (see next slide)

Suggestions to Update the MLSE COM Calculations

1) Change symbol error rate to error event rate (DER)
2) Update the equations used in steps 2 to 5 as per slides 18 and 19 to include the effect of noise coloring
3) As a part of ongoing improvement, adopt the approach to individually obtain each individual PDF of the MLSE sequence noise for every sequence length ($PDF_{noise,jEE}$) and since $PDF_{noise,jEE}$ is not computable for colored non-Gaussian noise, borrow the convolution formula from white non-Gaussian noise (slide 16) and scale its argument by the ratio between $\sigma_{noise,jEE}$ for white noise and colored noise to mimic the effect of noise coloring to obtain:

$$DER_{MLSE} \approx 2\sum_{j=1}^{\infty}\left(\frac{L-1}{L}\right)^{j}\left(1 - CDF_{noise,jEE}\left(\frac{main}{L-1}\frac{\left(\mathrm{trace}(\rho_{noise,jEE})\right)^{\frac{3}{2}}}{\sqrt{\sum_{vertical}\sum_{horizental}(\rho_{noise,jEE})}}\right)\right)$$

Changes to the Steps

1) Step 1 doesn't change
2) Calculation of SNR_{DFE} in step 2 doesn't change

   Change DER_{DFE} in step 2 to (Appendix A):

$$DER_{DFE} \approx 2\frac{L-1}{L}\left(1 - CDF_{noise}\left(\frac{main}{L-1}\right)\right)$$

3) Change DER_{MLSE} in step 3 to:

$$DER_{MLSE} \approx 2\sum_{j=1}^{\infty}\left(\frac{L-1}{L}\right)^{j}\left(1 - CDF_{noise,jEE}\left(\frac{main}{L-1}\frac{\left(\mathrm{trace}(\rho_{noise,jEE})\right)^{\frac{3}{2}}}{\sqrt{\sum_{vertical}\sum_{horizental}(\rho_{noise,jEE})}}\right)\right)$$

This change is for switching from symbol error rate to decision error ratio

This change is for switching from symbol error rate to decision error ratio and adding noise coloring effect

where $CDF_{noise,jEE}$ is calculated from:

$$PDF_{noise,jEE}(x) = PDF_{noise}(x) * \mathrm{conv}_{i=2}^{j} PDF_{noise}\big((1-\alpha)x\big) * PDF_{noise}(\alpha x)$$

and $\rho_{noise,jEE}$ is calculated from the correlation coefficients of the noise:

$$\rho_{noise,jEE} = \begin{bmatrix} 1 & -(1-\alpha)\rho_1 & +(1-\alpha)\rho_2 & \cdots & (-1)^{j+1}\alpha\rho_j \\ -(1-\alpha)\rho_{-1} & (1-\alpha)^2 & -(1-\alpha)^2\rho_1 & \cdots & (-1)^{j}\alpha(1-\alpha)\rho_{j-1} \\ +(1-\alpha)\rho_{-2} & -(1-\alpha)^2\rho_{-1} & (1-\alpha)^2 & \cdots & (-1)^{j-1}\alpha(1-\alpha)\rho_{j-2} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ (-1)^{j+1}\alpha\rho_{-j} & (-1)^{j}\alpha(1-\alpha)\rho_{-(j-1)} & (-1)^{j-1}\alpha(1-\alpha)\rho_{-(j-2)} & \cdots & \alpha^2 \end{bmatrix}$$

Changes to the Steps

which are calculated from the overall colored noise PSD:

$$R_{NN}(\tau) = F^{-1}\{PSD_{noise}\}$$

which is obtained as power sum of the individual noise PSDs, each calculated based on their corresponding shaping filters (see slide 23 as an example)

If in addition any of the individual input noise sources is also colored, modify COM to import its PSD and put it through its noise shaping filter

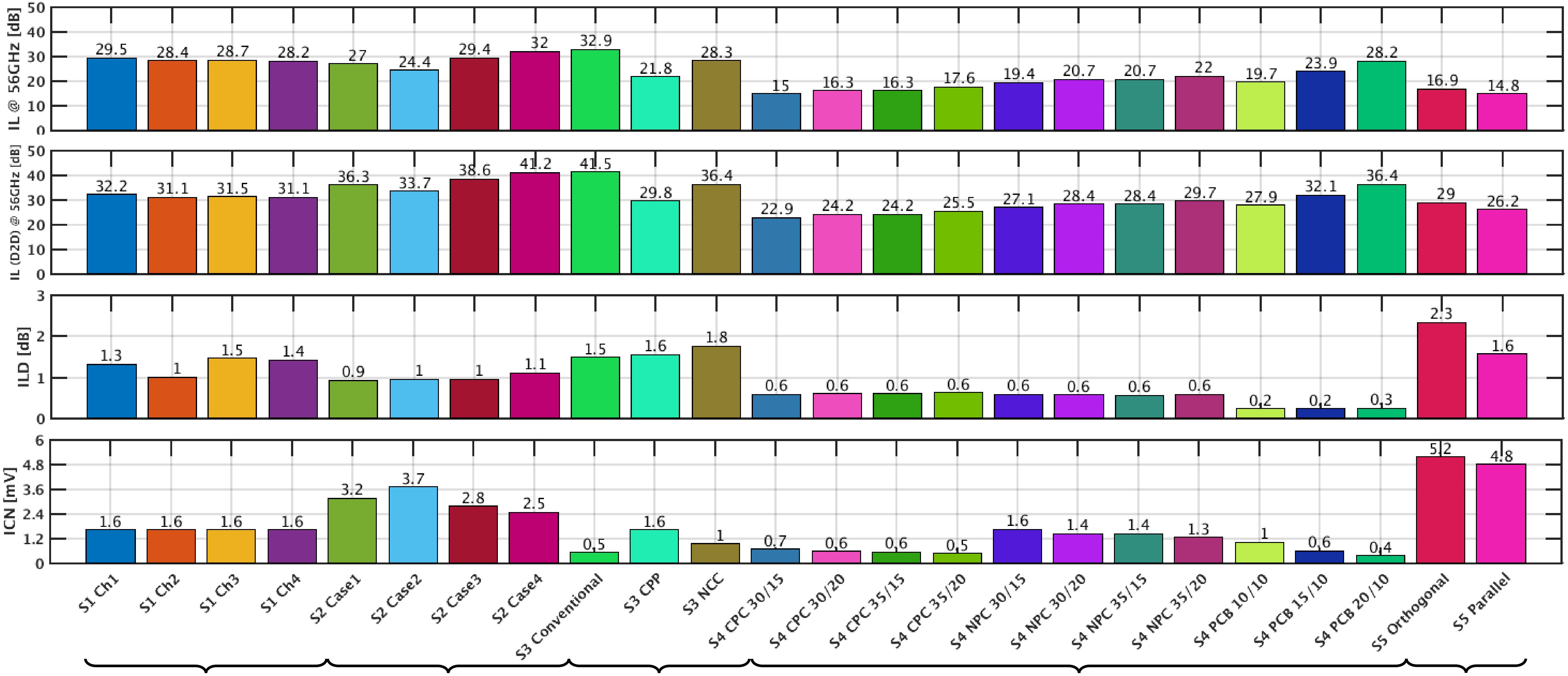
4) Change $SNR_{DFE,equivalent}$ in step 4 to (Appendix B):

This change is to reflect the above changes

$$SNR_{DFE,equivalent} = \left(\frac{L-1}{main} CDF_{noise}^{-1}\left(1 - \frac{1}{2}\frac{L}{L-1} DER_{MLSE}\right)\right)^2 SNR_{DFE}$$

5) Step 5 doesn't change

24 Study Cases



S1 **S2:** oif2022.194.00 **S3:** oif2022.313.02 **S4:** oif2022.470.00 **S5:** rabinovich_3dj_01_230116

Study Cases – Link Parameters

- Fix parameters were taken from the original channel documents
- Parameters that needed optimization were optimized using proprietary tool

Channel	Bit Rate [Gb/s]	Thru Swing [mV]	Fext Swing [mV]	Next Swing [mV]	TX FIR [Pre / Post]	Die C_d [fF] L_s [pH]	C_b [fF]	Package [mm] [Ω]	Rx Filter BW	CTLE Pole/Zero Ratio	DFE [# of Taps]	Rx FFE [Pre / Post]	TX SNR [dB]	Rx Noise [V^2/GHz]	Jitter Rand / DD [UI]
S1	224	413	413	608	3 / 1	40/90/110 130/150/140	Included In channel	Included In channel	0.75 x f_b	80/2.5/1	1	6 / 8	32.5	4.1E-8	0.01 / 0.02
S2	224	442	442	608	3 / 1	40/90/110 130/150/140	30	30 92.5	0.75 x f_b	100/2.5/1	1	0 / 24	33	4.1E-8	0.01 / 0.02
S3	224	413	413	608	3 / 1	40/90/110 130/150/140	30	30 92.5	0.75 x f_b	80/2.5/1	1	0 / 24	33	4.1E-8	0.01 / 0.02
S4	224	413	413	608	3 / 1	40/90/110 130/150/140	40	30 92.5	0.75 x f_b	80/2.5/1	1	0 / 24	33	4.1E-8	0.01 / 0.02
S5	224	387	387	608	3 / 1	40/90/110 130/150/140	30	45 / 0 92	0.75 x f_b	100/2.5/1	1	0 / 8	32.5	4.1E-9	0.01 / 0.02

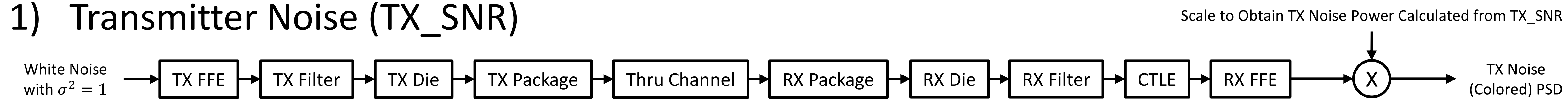
Study Cases – Summary of Results

Channel	Variant	DFE Tap $= \alpha$	Theoretical Coding Gain [dB]	DER				$\Delta SNR = \Delta COM$ [dB]		
				DFE	MLSE (White)	MLSE (Color)	Color / White	MLSE (White)	MLSE (Color)	Color - White
S1	Channel 1	0.8116	2.1977	2.3368 E-9	2.6393 E-14	2.9647 E-14	1.1233	2.2915	2.2784	-0.0130
	Channel 2	0.7272	1.8437	4.3729 E-10	1.4863 E-14	6.1617 E-15	0.4146	2.0043	2.1070	0.1027
	Channel 3	0.7655	2.0029	1.2885 E-8	1.4157 E-12	1.0233 E-12	0.7228	2.1359	2.1875	0.0517
	Channel 4	0.7850	2.0849	1.9631 E-9	3.7920 E-14	3.8648 E-14	1.0192	2.2225	2.2198	-0.0027
S2	Case 1 *	0.8600	2.4046	2.5379 E-10	0	0	NA	NA	NA	NA
	Case 2 *	0.8894	2.5310	4.6791 E-12	0	0	NA	NA	NA	NA
	Case 3	0.8702	2.4485	1.1886 E-8	1.1725 E-13	1.9379 E-13	1.6528	2.5270	2.4520	-0.0750
	Case 4	0.8535	2.3768	6.2446 E-7	1.7034 E-10	1.8271 E-10	1.0726	2.4734	2.4596	-0.0139
S3	Conventional	0.9729	2.8924	5.8797 E-4	8.4273 E-6	1.5780 E-5	1.8725	2.7981	2.5626	-0.2355
	CPP	1.0000	3.0102	3.4236 E-6	2.7366 E-10	5.4181 E-10	1.9798	2.8520	2.7198	-0.1324
	NCC	0.9923	2.9768	3.7980 E-4	2.9809 E-6	6.1568 E-6	2.0654	2.9143	2.6592	-0.2551
S4	CPC 30/15 *	0.8389	2.3141	2.1982 E-14	0	0	NA	NA	NA	NA
	CPC 30/20 *	0.8361	2.3022	4.1267 E-13	0	0	NA	NA	NA	NA
	CPC 35/15 *	0.8388	2.3136	3.9835 E-13	0	0	NA	NA	NA	NA
	CPC 35/20 *	0.9843	2.9419	8.7550 E-12	0	0	NA	NA	NA	NA
	NPC 30/15	0.9819	2.9315	5.5859 E-8	1.4547 E-13	1.5819 E-12	10.8746	2.8713	2.5082	-0.3630
	NPC 30/20	0.9847	2.9439	4.6879 E-7	8.4511 E-12	5.8129 E-11	6.8783	2.8930	2.5498	-0.3432
	NPC 35/15	0.9850	2.9452	4.7215 E-7	8.5130 E-12	5.8570 E-11	6.8801	2.8943	2.5509	-0.3434
	NPC 35/20	0.9837	2.9397	1.9817 E-6	1.3860 E-10	7.4913 E-10	5.4049	2.8809	2.5444	-0.3366
	PCB 10/10 *	0.9906	2.9693	3.3622 E-9	0	2.6229 E-15	NA	NA	2.6427	-0.1022
	PCB 15/10	0.9815	2.9300	5.6901 E-7	1.3188 E-11	6.5554 E-11	4.9707	2.9108	2.6185	-0.2922
	PCB 20/10	0.9542	2.8113	5.5608 E-5	1.2779 E-7	2.6143 E-7	2.0458	2.7758	2.5738	-0.2020
S5	Orthogonal	0.9182	2.6555	3.4296 E-8	2.8283 E-13	7.7534 E-13	2.7413	2.5895	2.4404	-0.1491
	Parallel	0.8625	2.4151	7.5850 E-9	6.2132 E-14	1.4243 E-13	2.2923	2.4505	2.3331	-0.1174

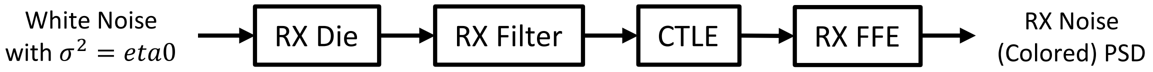
* Result are subject to numerical inaccuracy

Study Cases – Noise Shaping Filters

1) Transmitter Noise (TX_SNR)

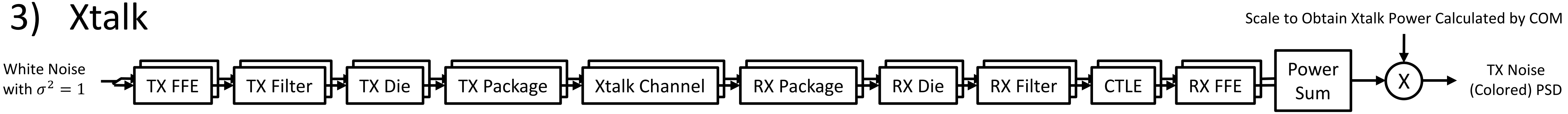


2) Receiver Noise (eta_0)

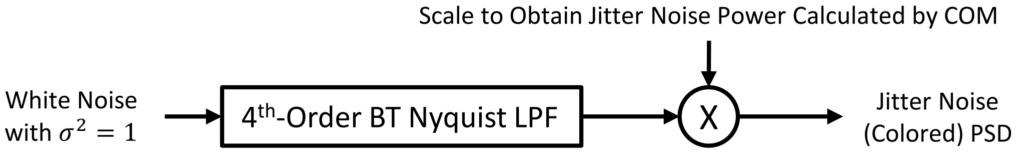


If receiver input noise is not white, its PSD should be used instead of the eta0 white noise

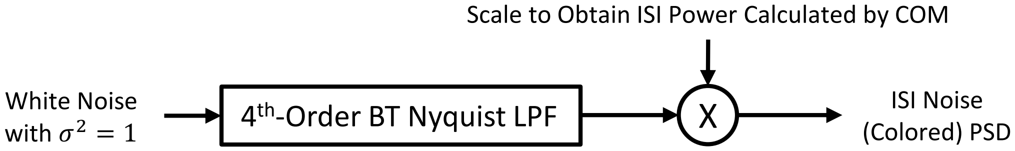
3) Xtalk



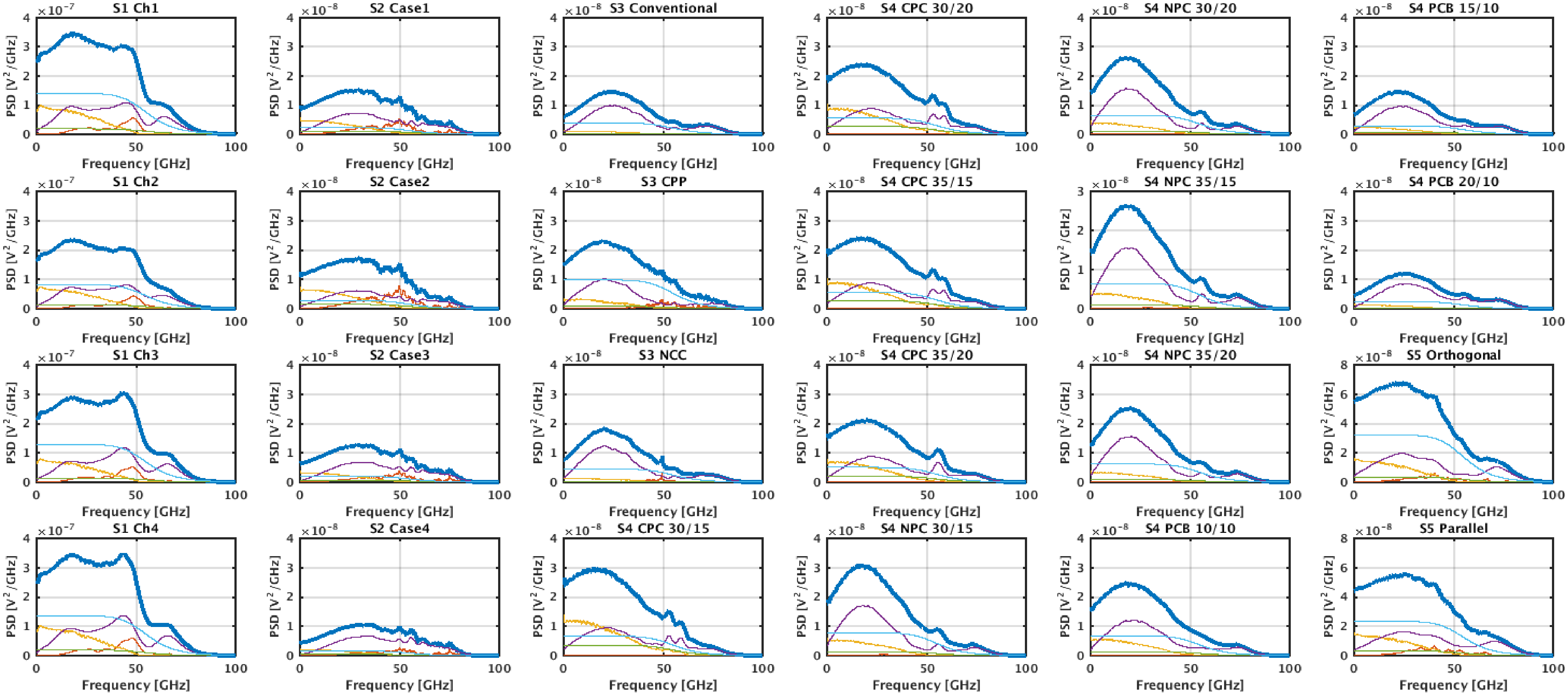
4) Jitter



5) ISI

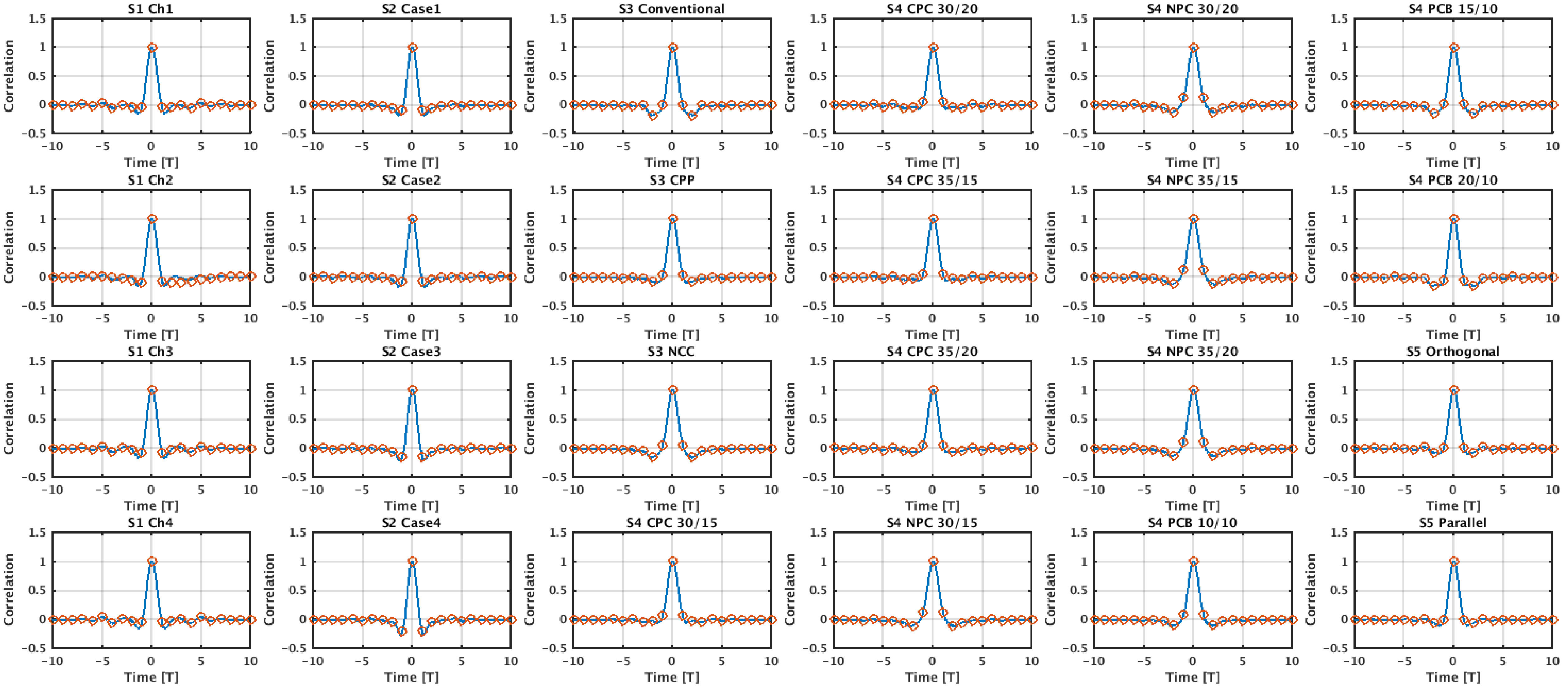


Noise PSD

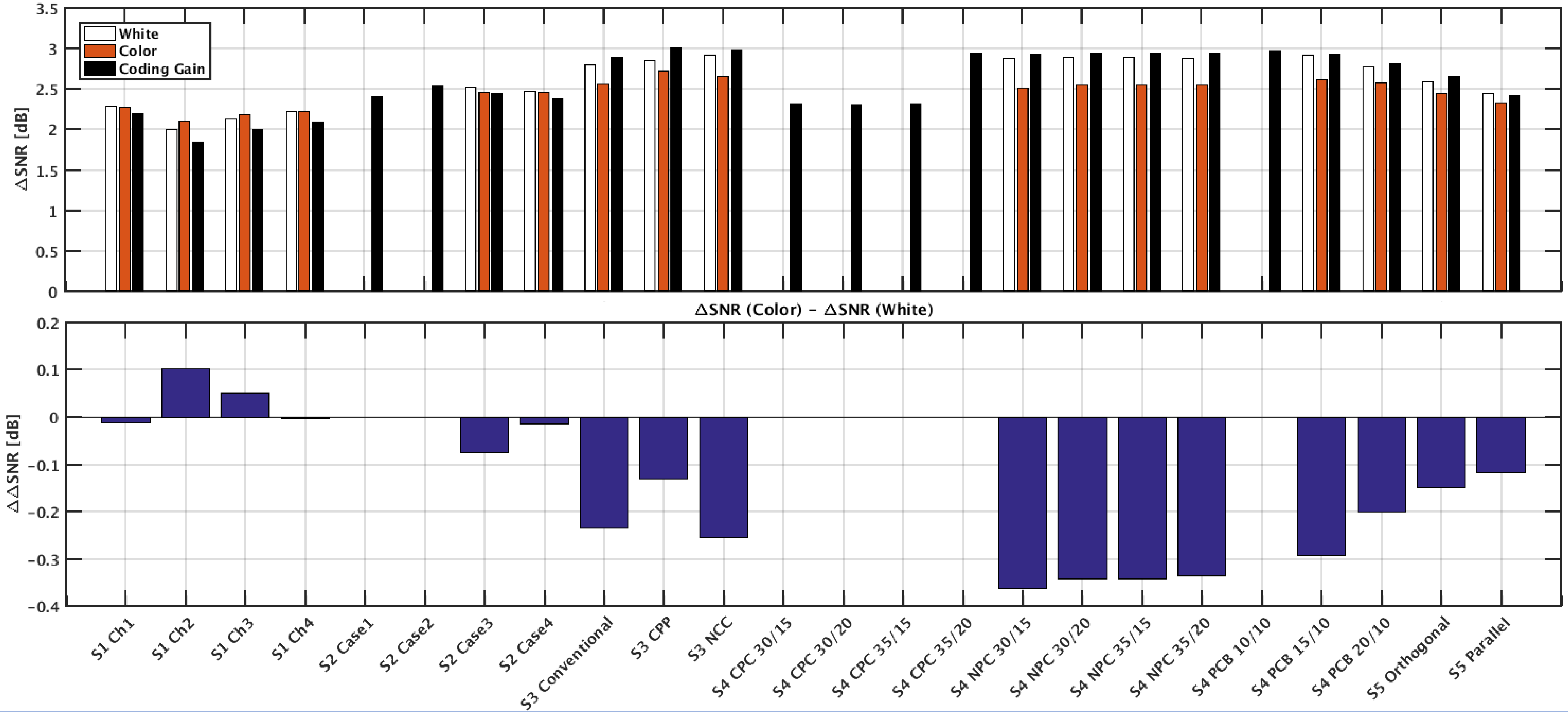


S1 Ch1
S2 Case1
S3 Conventional
S4 CPC 30/20
S4 NPC 30/20
S4 PCB 15/10
S1 Ch2
S2 Case2
S3 CPP
S4 CPC 35/15
S4 NPC 35/15
S4 PCB 20/10
S1 Ch3
S2 Case3
S3 NCC
S4 CPC 35/20
S4 NPC 35/20
S5 Orthogonal
S1 Ch4
S2 Case4
S4 CPC 30/15
S4 NPC 30/15
S4 PCB 10/10
S5 Parallel
×10⁻⁷
×10⁻⁸
PSD [V²/GHz]
Frequency [GHz]

Noise Correlation



MLSE COM (ΔSNR) and DER Comparison



Summary and Conclusions

- Analysis of error event probability of MLSE under colored noise was performed
- MLSE COM (ΔSNR) calculation was repeated for colored noise
- $PDF_{noise,jEE}$ formula was upgraded to obtain individual PDFs for each error event length
- Noise coloring can increase or decrease total noise along the sequence depending on the correlation between noise samples and α
- Wireline channels have different noise injection points with noises with different PSDs and shaped by different filter responses
- As a result, coloring has a compound effect that should be evaluated case by case
- If input noise is pre-colored method works but COM needs to have capability to import its PSD
- For study cases (17 yielded out of 24 cases), MLSE COM loss due to noise coloring averaged at -0.16dB and within -0.36dB to 0.10dB
- **It is recommended that MLSE COM equations be updated to incorporate:**
  - **Switching from symbol error rate to event error rate (decision error ratio)**
  - **Effect of noise coloring**
  - **Individual $PDF_{noise,jEE}$ for each error event length**
- **It is also recommended that importing input noise PSD capability be added to COM tool**

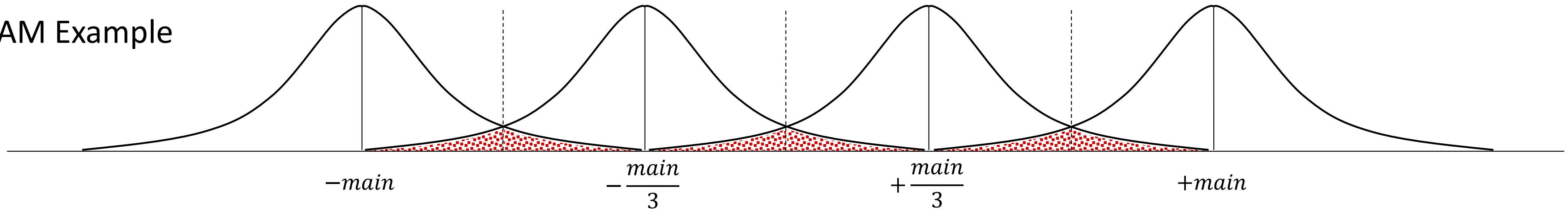
Appendix A

Analysis of L-PAM 1-Tap DFE

Error Event Rate (Decision Error Ratio – DER)

- Assuming outer PAM levels of $\pm main$ and dominance of adjacent-level errors

4-PAM Example



- Error event probability (no error propagation)

$$DER_{DFE} \approx 2\frac{L-1}{L}\left(1 - CDF_{noise}\left(\frac{main}{L-1}\right)\right)$$

- This is the new equation for step 2

Appendix B

Analysis of the Conceptual Equivalent DFE

SNR

$$DER_{DFE} \approx 2\frac{L-1}{L}Q\left(\sqrt{\frac{3}{L^2-1}SNR_{DFE,equivalent}}\right)$$

$$DER_{DFE,equivalent} = DER_{MLSE}$$

$$SNR_{DFE,equivalent} = \left(\frac{L-1}{main}CDF_{noise}^{-1}\left(1-\frac{1}{2}\frac{L}{L-1}DER_{MLSE}\right)\right)^2 SNR_{DFE}$$

- This is the new equation for step 4