Error Propagation Analysis of MLSE

Hossein Shakiba
Affiliated with Huawei Technologies Canada
April 20, 2023

HUAWEI

Outline

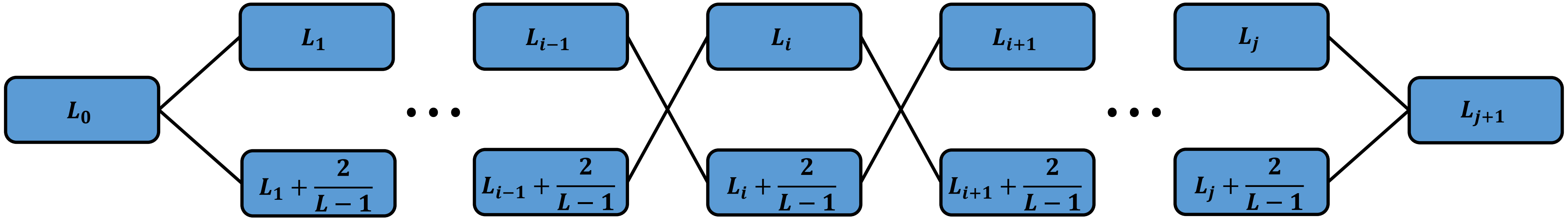
- Motivation and Background
- Conditional Probability of a Burst
- Error Propagation Factor and Average Burst Length
- Two Extreme Conceptual examples
  - Comparisons to DFE
  - Simplifications for White Gaussian Noise
- 24 Study Cases
  - Link Parameters
  - Summary of Results
  - CDF of Error Burst
  - PDF of Error Burst
  - EPF
  - Error Burst Length
  - Error Burst Length against Spec
- Summary and Conclusions

Motivation

- Error propagation in MLSE is not nearly as known as in DFE
- Noise coloring makes this more convoluted
- This contribution describes an attempt to analyze and statistically model MLSE error propagation in a similar manner to the well known EPP model of the DFE
- At this time this contribution is only for awareness and is not proposing any specific change or direction
- If there is interest to turn the results into an action, more study is recommended particularly from the aspects of:
  - Model validation by means of independent studies and simulations
  - FEC analysis (e.g. statistical) based on the developed MLSE error propagation model
  - More ideas …

Background

- Contribution shakiba_3df_01b_2211 showed that error events of an L-PAM $1+\alpha D$ MLSE are dominated by a zig-zag pattern in the form of alteranting adjacent levels:



$$L_i \in \begin{cases} -1,-1+\frac{2}{L-1},\cdots,+1-\frac{2}{L-1},+1 & i=0,j+1 \\ -1,-1+\frac{2}{L-1},\cdots,+1-\frac{2}{L-1} & i=1,\ldots,j \end{cases}$$

- Contribution shakiba_3dj_01_230420 calculated the probability of a j-error event, an error event that causes a burst of j errors:

$$P(B_{MLSE}=j) \approx 2\left(\frac{L-1}{L}\right)^j\left(1-CDF_{noise,jEE}\left(\frac{main}{L-1}\frac{\left(\mathrm{trace}(\rho_{noise,jEE})\right)^{\frac{3}{2}}}{\sqrt{\Sigma_{vertical}\,\Sigma_{horizental}(\rho_{noise,jEE})}}\right)\right)$$

Conditional Probability of a Burst

- As a result the conditional probability of a burst can be calculated:

$$P(B_{MLSE} = j|Error\ Event) = \frac{P(B_{MLSE}=j)}{\sum_j P(B_{MLSE}=j)} \approx \frac{2\left(\frac{L-1}{L}\right)^j \left(1-CDF_{noise,jEE}\left(\frac{main}{L-1}\frac{\left(\mathrm{trace}\left(\rho_{noise,jEE}\right)\right)^{\frac{3}{2}}}{\sqrt{\sum_{vertical}\sum_{horizental}\left(\rho_{noise,jEE}\right)}}\right)\right)}{2\sum_{j=1}^{\infty}\left(\frac{L-1}{L}\right)^j \left(1-CDF_{noise,jEE}\left(\frac{main}{L-1}\frac{\left(\mathrm{trace}\left(\rho_{noise,jEE}\right)\right)^{\frac{3}{2}}}{\sqrt{\sum_{vertical}\sum_{horizental}\left(\rho_{noise,jEE}\right)}}\right)\right)}$$

Error Propagation Factor and Average Burst Length

- Similar to DFE, we can define an Error Propagation Factor (EPF) for MLSE:

$$EPF_{MLSE}(\alpha,\sigma_{noise},j)=\frac{P(B_{MLSE}=j+1|Error\ Event)}{P(B_{MLSE}=j|Error\ Event)}\approx\frac{L-1}{L}\frac{1-CDF_{noise,(j+1)EE}\left(\frac{main}{L-1}(1+j(1-\alpha)^2+\alpha^2)\right)}{1-CDF_{noise,jEE}\left(\frac{main}{L-1}(1+(j-1)(1-\alpha)^2+\alpha^2)\right)}\quad,j=1,2,\cdots$$

- However, EPF_{MLSE} is in general a function of j (burst length) and unlike DFE, error propagation of MLSE cannot be statistically modeled with a simple exponential distribution
  - Recall for DFE: $EPF_{DFE}(\alpha,\sigma_{noise})=EPP_{DFE}(\alpha,\sigma_{noise})=P(Next\ Error|Current\ Error)$
- For calculating average burst length symbol error rate is needed:

$$SER_{MLSE}\approx 2\sum_{j=1}^{\infty}j\left(\frac{L-1}{L}\right)^j\left(1-CDF_{noise,jEE}\left(\frac{main}{L-1}(1+(j-1)(1-\alpha)^2+\alpha^2)\right)\right)$$

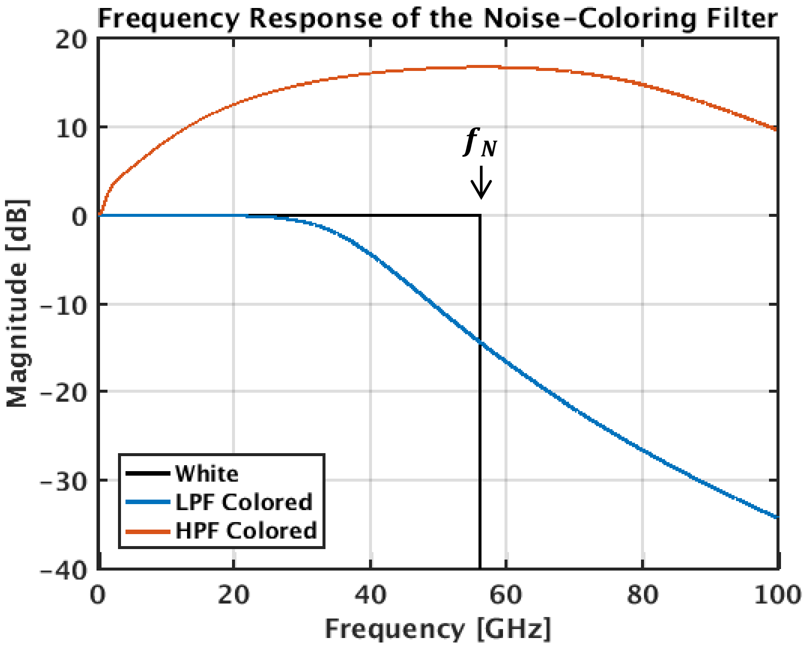
which results in an average burst length of:

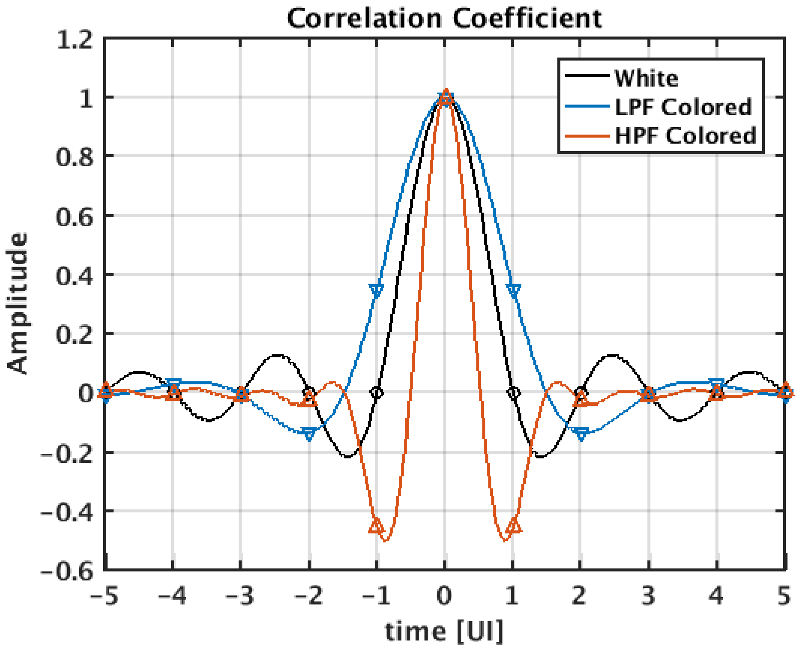
$$\bar{B}_{MLSE}(\alpha,\sigma_{noise})=\frac{SER_{MLSE}}{DER_{MLSE}}\approx\frac{\sum_{j=1}^{\infty}j\left(\frac{L-1}{L}\right)^j\left(1-CDF_{noise,jEE}\left(\frac{main}{L-1}(1+(j-1)(1-\alpha)^2+\alpha^2)\right)\right)}{\sum_{j=1}^{\infty}\left(\frac{L-1}{L}\right)^j\left(1-CDF_{noise,jEE}\left(\frac{main}{L-1}(1+(j-1)(1-\alpha)^2+\alpha^2)\right)\right)}$$

  - Recall for DFE $\bar{B}_{DFE}(\alpha,\sigma_{noise})$ is also equal to $\frac{1}{1-EPP_{DFE}(\alpha,\sigma_{noise})}$

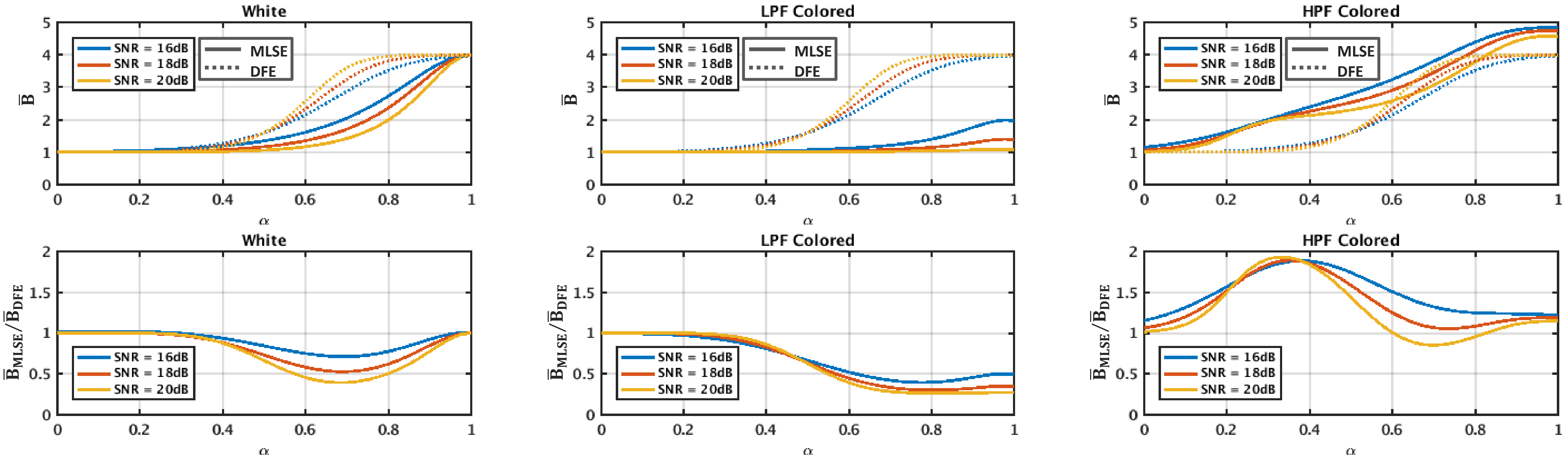
Two Extreme Conceptual Examples

- We continue the analysis by using two LPF and HPF noise coloring filters (same filters used in shakiba_3dj_01_230420.pdf) to demonstrate the effect of extreme noise coloring on the MLSE error propagation (Gaussian noise assumption)
- Note that to explore trends and limits these cases are extreme and non-real as in real cases noise is always a combination of several components, each colored differently





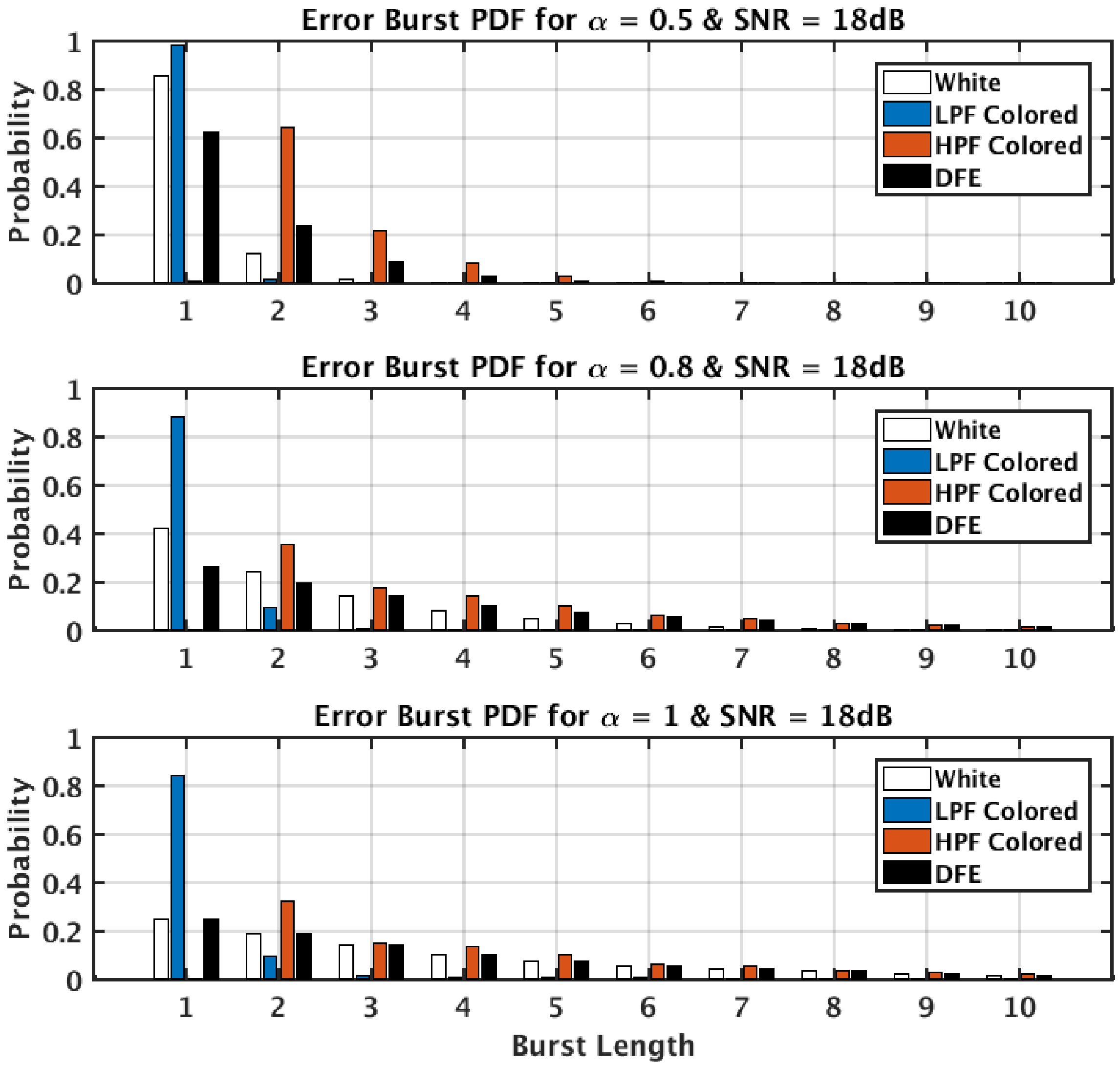
Conceptual Examples and Comparisons to DFE



- Consider more practical range of $0.5 < \alpha \leq 1$
- With no coloring average burst length is always shorter than DFE
- With LPF coloring average burst length is always much shorter than DFE
- With HPF coloring, depending on α and SNR, average burst length could become shorter (higher SNR) or longer (lower SNR) than DFE

Conceptual Examples and Comparisons to DFE

- White noise:
  - MLSE error propagation is always better than DFE and approaches DFE as $\alpha \rightarrow 1$
- LPF coloring:
  - Bursts rarely occur
  - MLSE error propagation is always much better than DFE
  - Even at high α values most of the errors are single and probability of longer bursts very quickly reduces
- HPF coloring:
  - Single errors rarely occur
  - MLSE error propagation depending on α and SNR could become better (higher SNR) or worse (lower SNR) than DFE (previous slide)
  - Worst error propagation of MLSE is more concentrated around shorter bursts



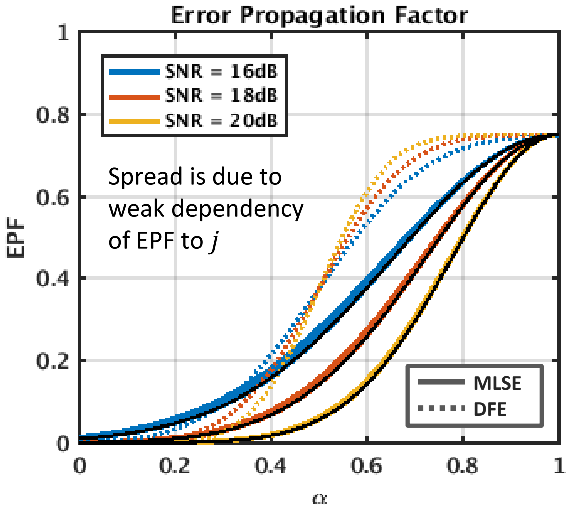
Simplifications for White Gaussian Noise

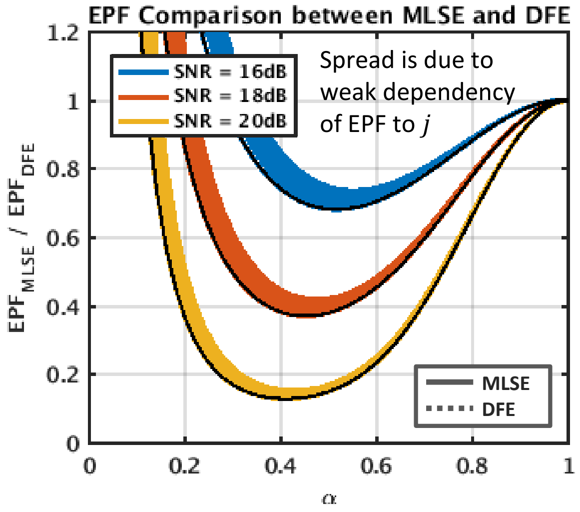
- In the case of white Gaussian noise it can be shown that the dependency of EPF_{MLSE} to burst length reduces and becomes a single probability:

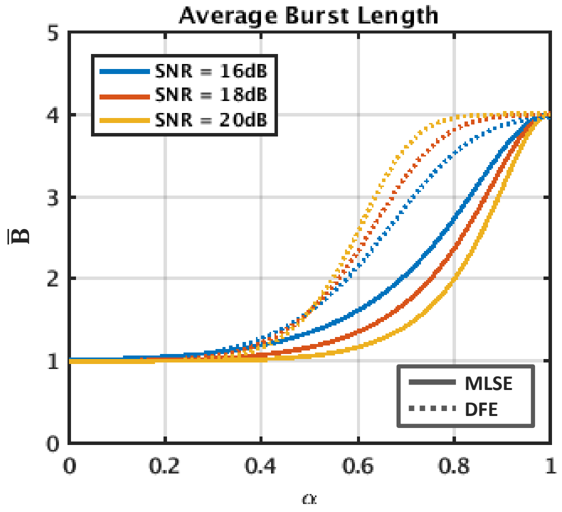
$$EPF_{MLSE}(\alpha,\sigma_{noise},j) \approx EPF_{MLSE}(\alpha,\sigma_{noise}) = EPP_{MLSE}(\alpha,\sigma_{noise}) \approx \frac{L-1}{L}\frac{Q\left(\frac{main}{L-1}\frac{\sqrt{1+(1-\alpha)^2+\alpha^2}}{\sigma_{noise}}\right)}{Q\left(\frac{main}{L-1}\frac{\sqrt{1+\alpha^2}}{\sigma_{noise}}\right)} \quad ,White\ Gaussian$$

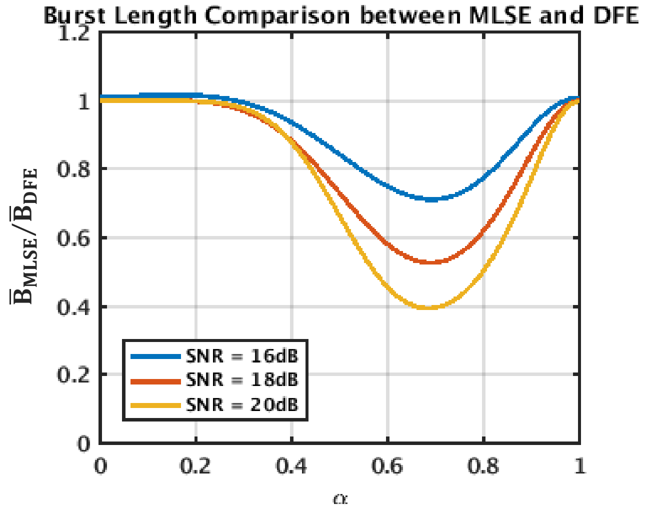
- The simple EPP approach of DFE can now be applied, resulting in an average burst length of:

$$\overline{B_{MLSE}}(\alpha,\sigma_{noise}) \approx \frac{1}{1-EPF_{MLSE}(\alpha,\sigma_{noise})} \quad ,White\ Gaussian$$

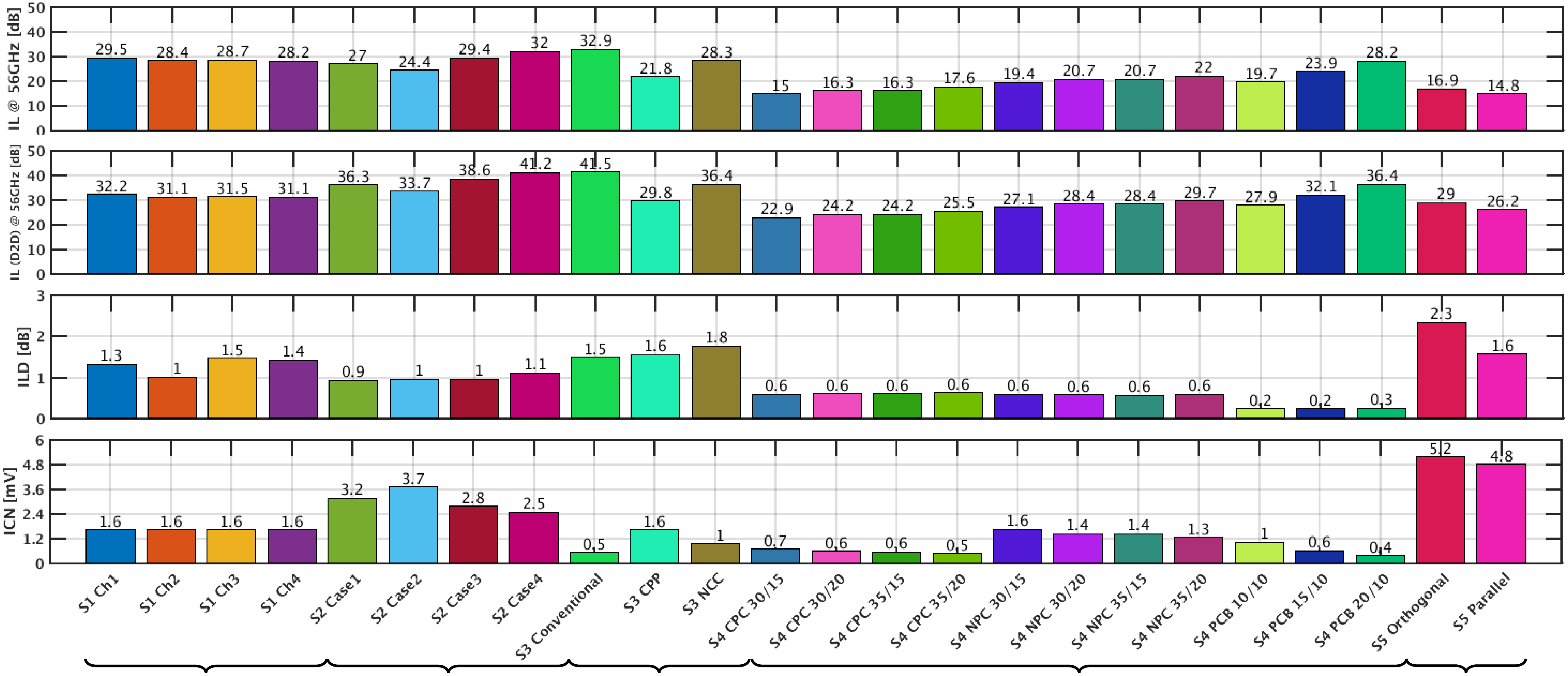








24 Study Cases



S1 **S2:** oif2022.194.00 **S3:** oif2022.313.02 **S4:** oif2022.470.00 **S5:** rabinovich_3dj_01_230116

Study Cases – Link Parameters

- Fix parameters were taken from the original channel documents
- Parameters that needed optimization were optimized using proprietary tool

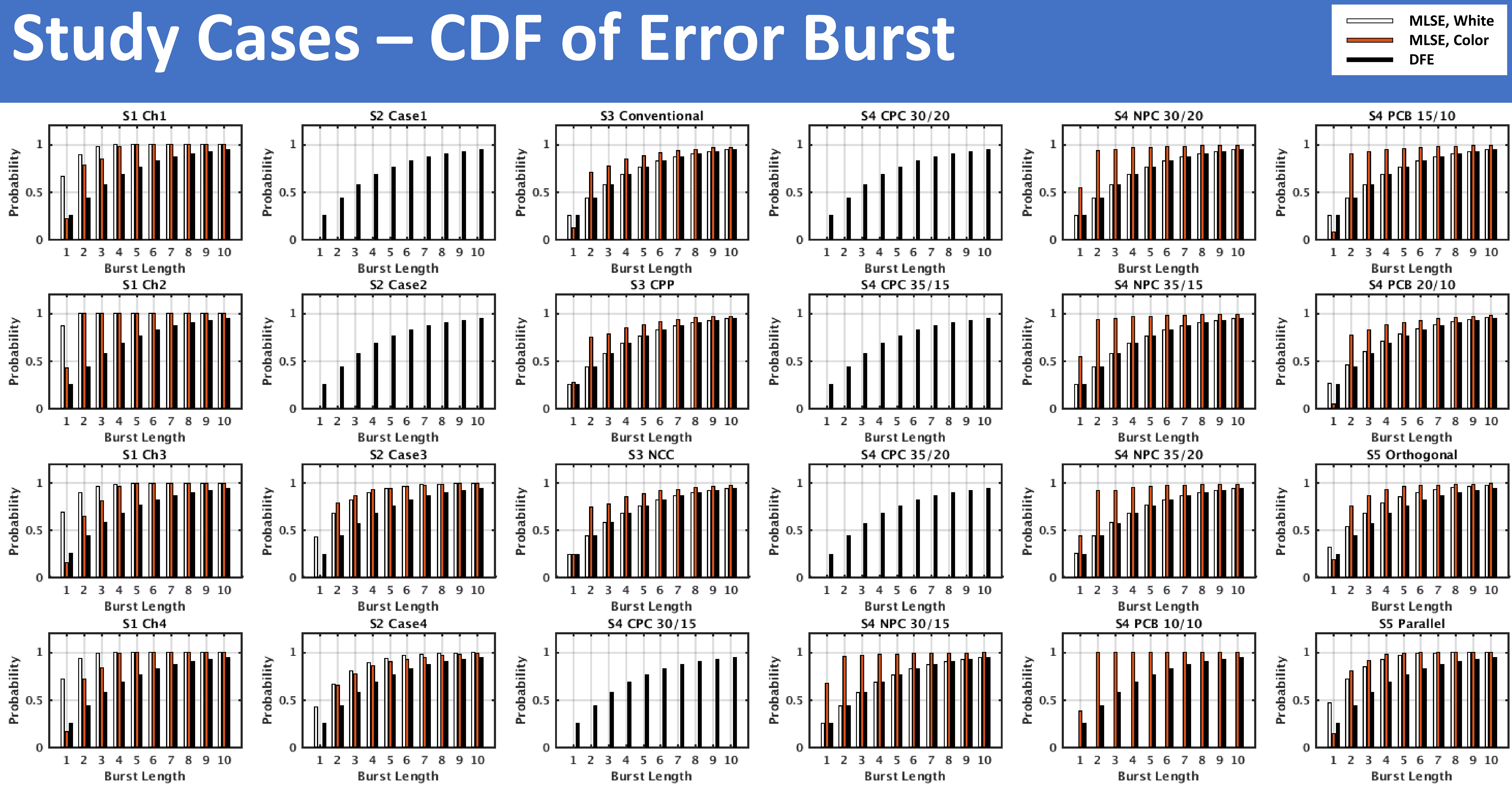
Channel	Bit Rate [Gb/s]	Thru Swing [mV]	Fext Swing [mV]	Next Swing [mV]	TX FIR [Pre / Post]	Die C_d [fF] L_s [pH]	C_b [fF]	Package [mm] [Ω]	Rx Filter BW	CTLE Pole/Zero Ratio	DFE [# of Taps]	Rx FFE [Pre / Post]	TX SNR [dB]	Rx Noise [V^2/GHz]	Jitter Rand / DD [UI]
S1	224	413	413	608	3 / 1	40/90/110 130/150/140	Included In channel	Included In channel	0.75 x f_b	80/2.5/1	1	6 / 8	32.5	4.1E-8	0.01 / 0.02
S2	224	442	442	608	3 / 1	40/90/110 130/150/140	30	30 92.5	0.75 x f_b	100/2.5/1	1	0 / 24	33	4.1E-8	0.01 / 0.02
S3	224	413	413	608	3 / 1	40/90/110 130/150/140	30	30 92.5	0.75 x f_b	80/2.5/1	1	0 / 24	33	4.1E-8	0.01 / 0.02
S4	224	413	413	608	3 / 1	40/90/110 130/150/140	40	30 92.5	0.75 x f_b	80/2.5/1	1	0 / 24	33	4.1E-8	0.01 / 0.02
S5	224	387	387	608	3 / 1	40/90/110 130/150/140	30	45 / 0 92	0.75 x f_b	100/2.5/1	1	0 / 8	32.5	4.1E-9	0.01 / 0.02

Study Cases – Summary of Results

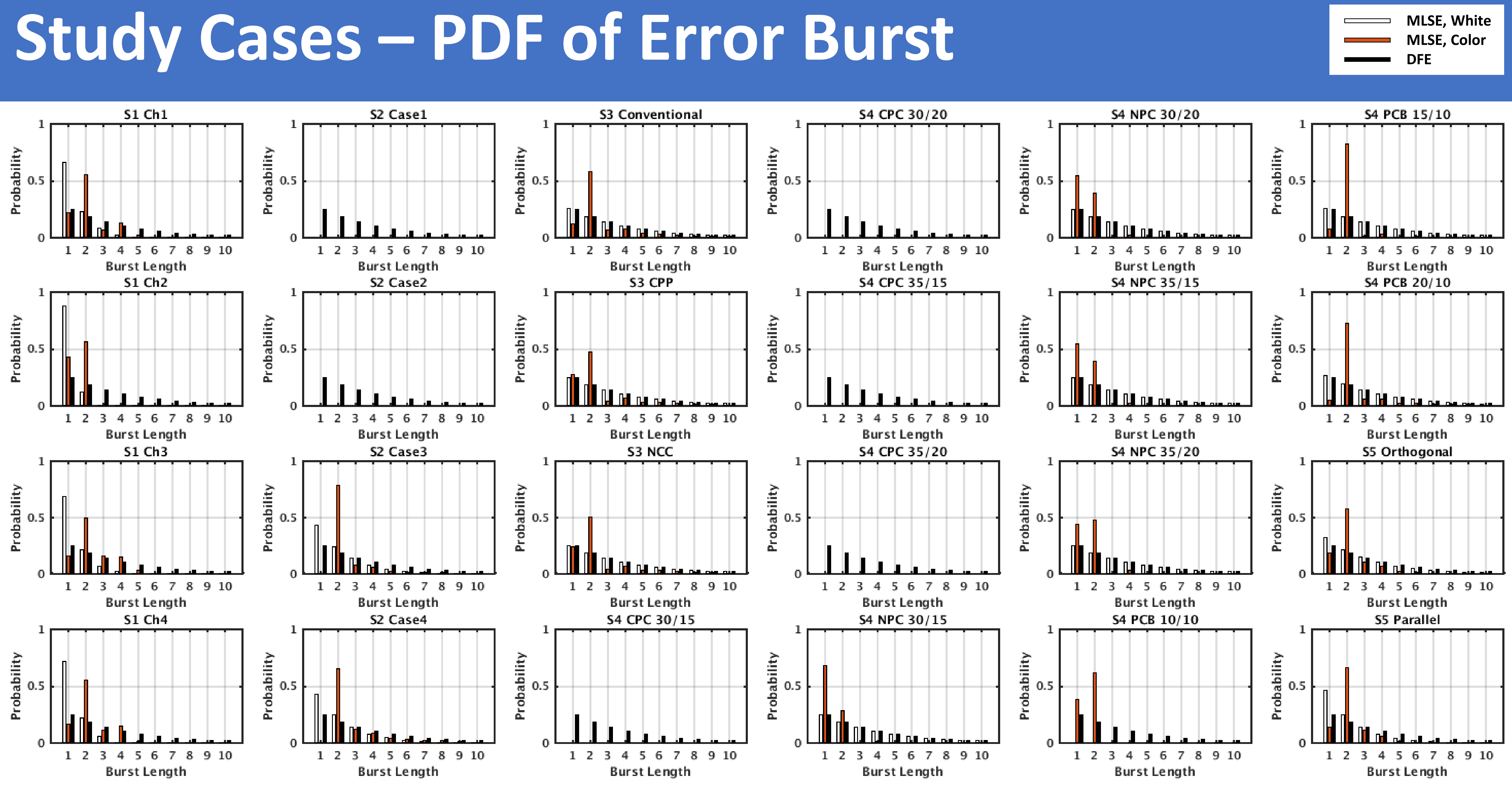
Channel	Variant	DFE Tap = α	Average Burst Length					
			DFE	MLSE (White)	MLSE (Color)	White/DFE	Color/DFE	Color / White
S1	Channel 1	0.8116	**3.9986**	1.4670	**2.1722**	0.3669	**0.5433**	1.4807
	Channel 2	0.7272	**3.9704**	1.1261	**1.5676**	0.2836	**0.3948**	1.3921
	Channel 3	0.7655	**3.9824**	1.4472	**2.4232**	0.3643	**0.6085**	1.6744
	Channel 4	0.7850	**3.9957**	1.3477	**2.2957**	0.3373	**0.5745**	1.7034
S2	Case 1 *	0.8600	**4.0000**	NA	**NA**	NA	**NA**	NA
	Case 2 *	0.8894	**4.0000**	NA	**NA**	NA	**NA**	NA
	Case 3	0.8702	**3.9998**	2.2740	**2.5503**	0.5685	**0.6376**	1.1215
	Case 4	0.8535	**3.9968**	2.3571	**3.0109**	0.5898	**0.7533**	1.2773
S3	Conventional	0.9729	**3.9901**	3.9536	**3.0038**	0.9908	**0.7528**	0.7598
	CPP	1.0000	**4.0000**	4.0000	**2.7930**	1.0000	**0.6982**	0.6982
	NCC	0.9923	**3.9960**	3.9967	**2.8188**	1.0002	**0.7054**	0.7053
S4	CPC 30/15 *	0.8389	**4.0000**	NA	**NA**	NA	**NA**	NA
	CPC 30/20 *	0.8361	**4.0000**	NA	**NA**	NA	**NA**	NA
	CPC 35/15 *	0.8388	**4.0000**	NA	**NA**	NA	**NA**	NA
	CPC 35/20 *	0.9843	**4.0000**	NA	**NA**	NA	**NA**	NA
	NPC 30/15	0.9819	**4.0000**	3.9589	**1.4911**	0.9897	**0.3728**	0.3766
	NPC 30/20	0.9847	**4.0000**	3.9759	**1.7302**	0.9940	**0.4325**	0.4352
	NPC 35/15	0.9850	**4.0000**	3.9768	**1.7307**	0.9942	**0.4327**	0.4352
	NPC 35/20	0.9837	**4.0000**	3.9724	**1.9201**	0.9931	**0.4800**	0.4834
	PCB 10/10 *	0.9906	**4.0000**	NA	**1.6190**	NA	**0.4048**	NA
	PCB 15/10	0.9815	**4.0000**	3.9565	**2.3201**	0.9891	**0.5800**	0.5864
	PCB 20/10	0.9542	**3.9979**	3.8139	**2.8469**	0.9540	**0.7121**	0.7465
S5	Orthogonal	0.9182	**4.0000**	3.1120	**2.3658**	0.7780	**0.5915**	0.7602
	Parallel	0.8625	**3.9998**	2.1016	**2.1778**	0.5254	**0.5445**	1.0362

* Result are subject to numerical inaccuracy

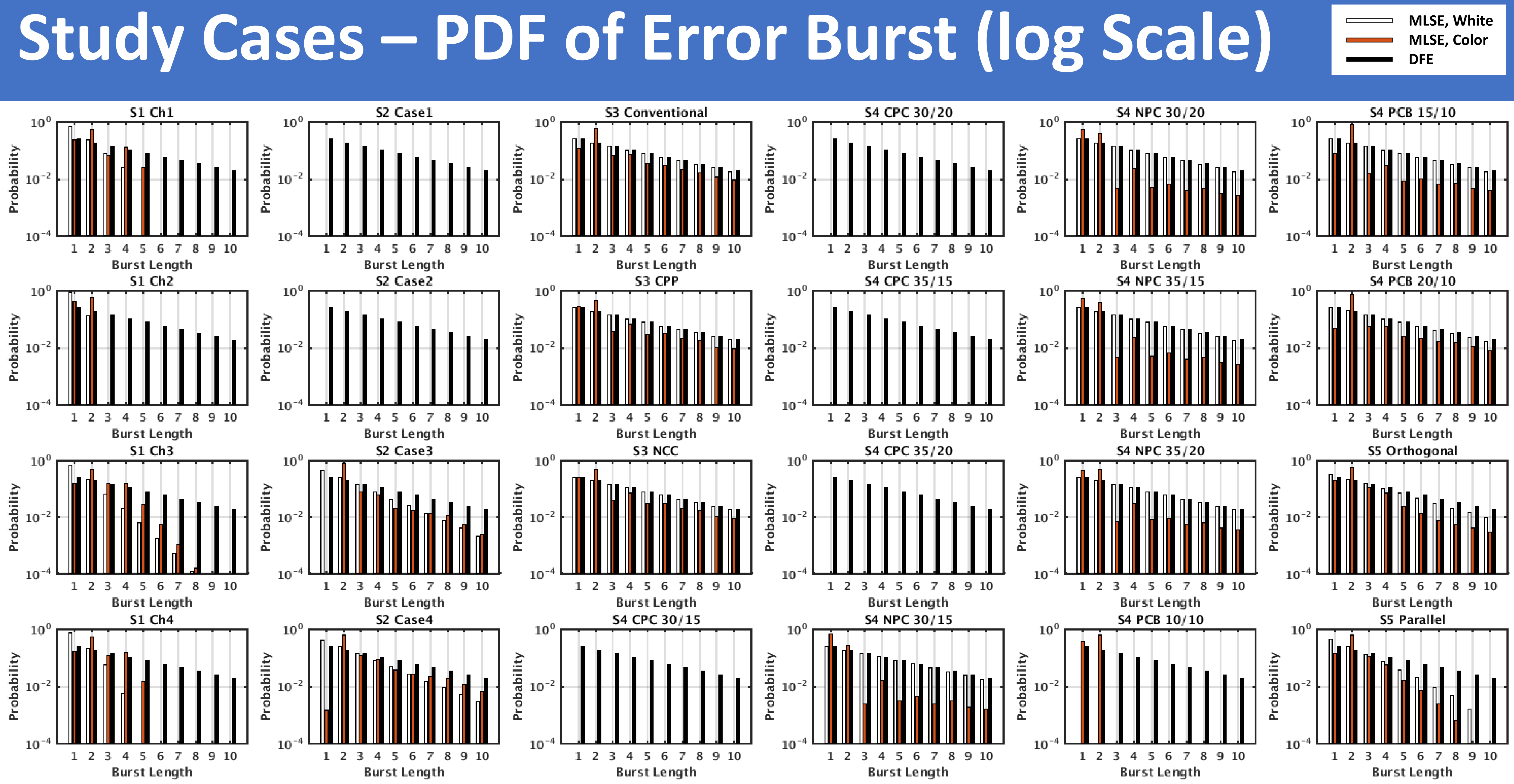
Study Cases – CDF of Error Burst



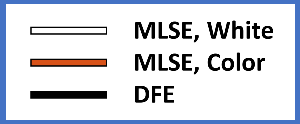
Study Cases – PDF of Error Burst

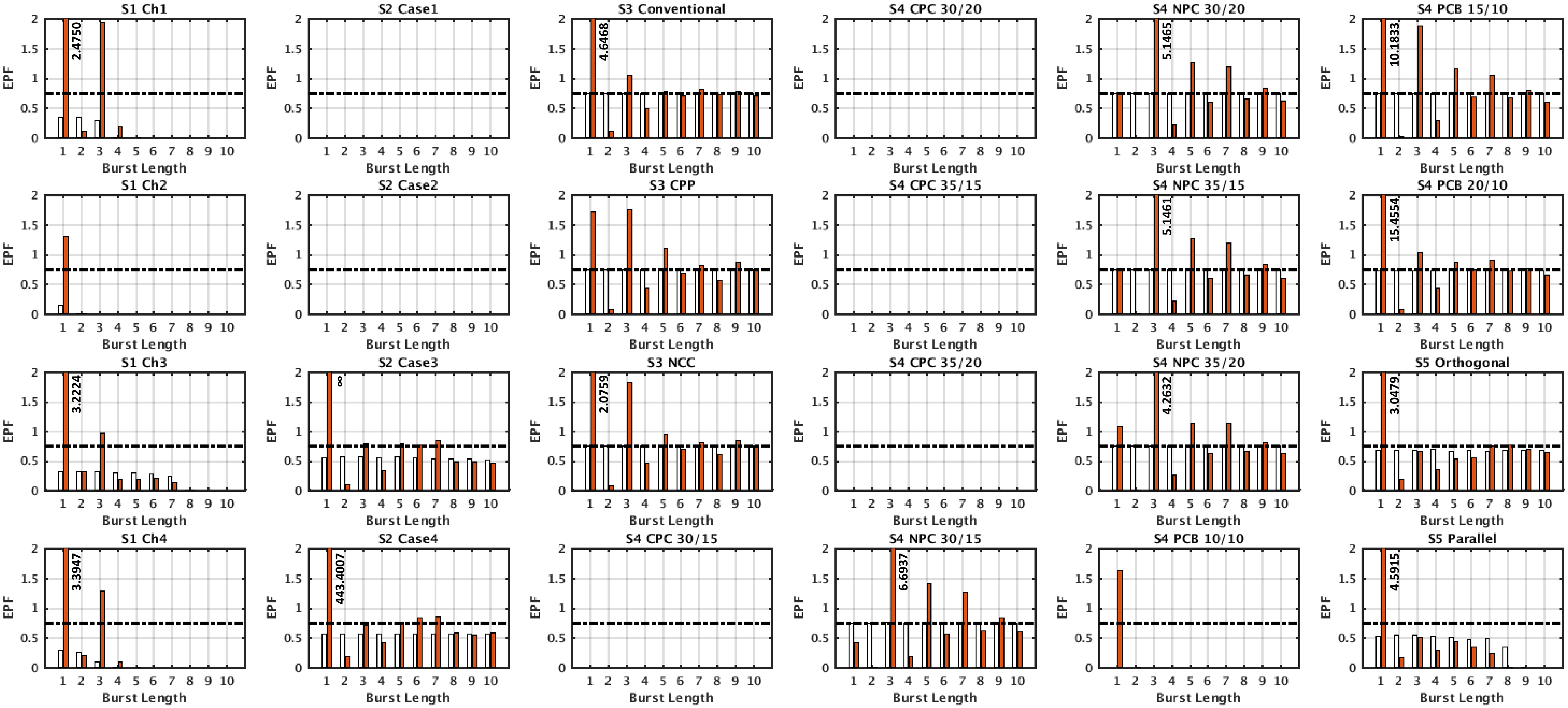


Study Cases – PDF of Error Burst (log Scale)



Study Cases – EPF

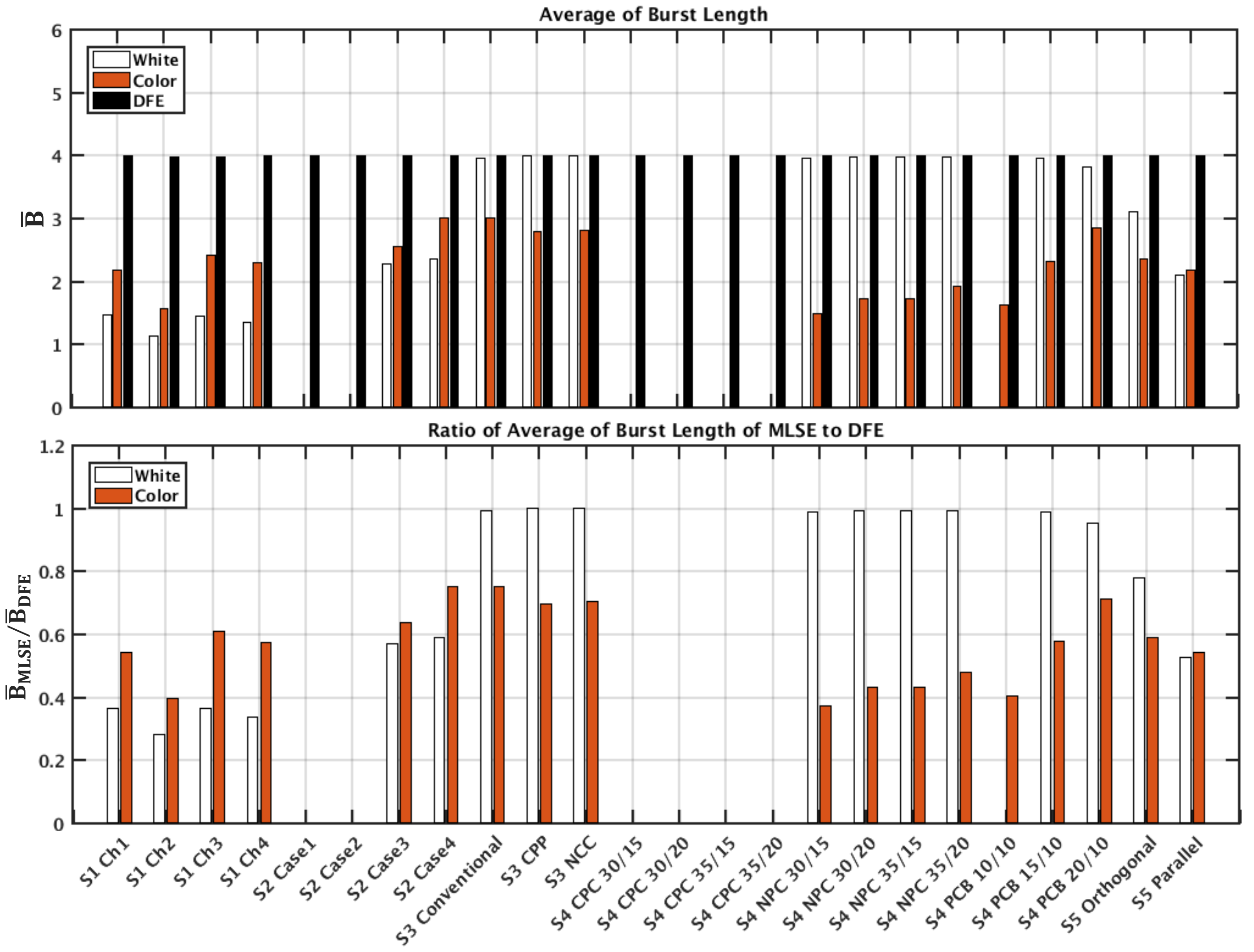




Study Cases – Error Burst Length

- For all cases, average error burst lengths of DFEs have maximized
- Average error burst lengths of MLSE with white noise are always same or less than DFE
- Average error burst lengths of MLSE with colored noise are always less than DFE
- On average, error burst length of MLSE with colored noise is noticeably the least

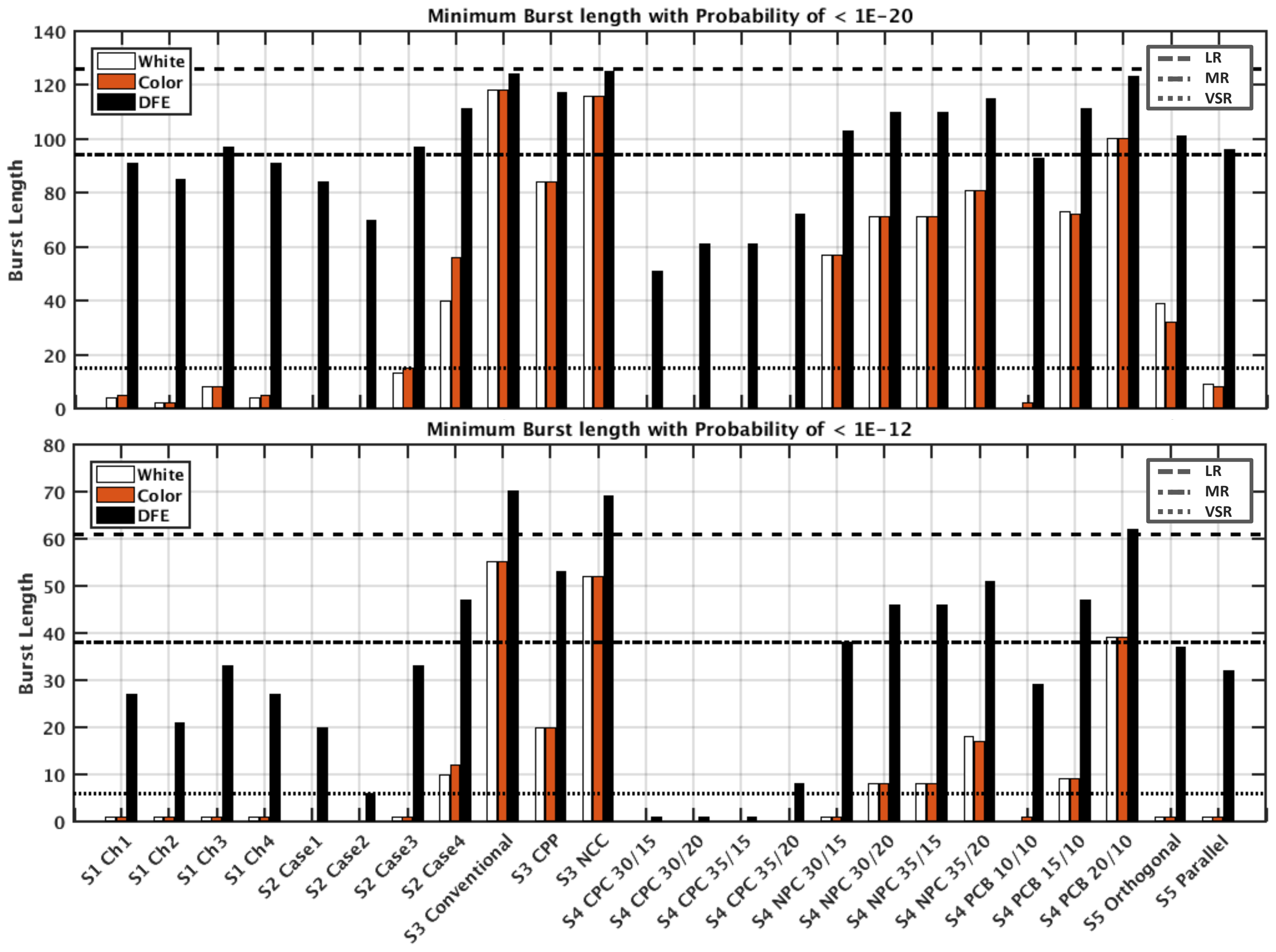
$\overline{B}$	DFE	MLSE	
		White	Color
Max	4.000	4.000	3.0109
Average	3.9970	2.9904	2.2687
Min	3.9704	1.1261	1.4911



Study Cases – Error Burst Length against (OIF) Spec

- Spec = Limit of burst lengths of such size that occur with such probability
- Few channels with DFE fail LR spec, most fail MR spec, and all fail VSR spec
- No channels with MLSE (with either white or colored noise) fail LR spec, few fail MR spec, and several fail VSR spec
- Noise coloring only slightly changes (+/-) long burst probabilities in MLSE

# (%) of Failing Channels	DFE	MLSE	
		White	Color
LR	3 (17.6%)	0 (0%)	0 (0%)
MR	14 (82.4%)	3 (17.6%)	3 (17.6%)
VSR	17 (100%)	11 (64.7%)	11 (64.7%)



Summary and Conclusions

- The following summary is based on analysis of 17 executable cases out of 24 examined cases
- Error propagation of MLSE, with or without noise coloring, always resulted in average shorter bursts compared to DFE (75% shorter for white noise and 57% shorter for colored noise)
- Error propagation of MLSE, with or without noise coloring, always resulted in a less probability of occurrence of longer bursts (> 5) compared to DFE
- Error propagation of MLSE without coloring approached DFE as $\alpha \rightarrow 1$ while coloring helped reduce longer burst probabilities
- Noise coloring caused a concentration of bursts around very short lengths (< 5 and e.g. clear observation and sometimes dominance of errors in pairs) and depending on the channel, could increase or decrease the probability of longer bursts
- On average, noise coloring reduced average burst lengths by 24%
- MLSE, with or without noise coloring, always resulted in less long bursts that are troubling the FEC compared to DFE, and was able to pass 100% / 78.6% / 35.3 % of the cases that failed the LR / MR / VSR burst length specs with DFE
- MLSE is better positioned to work with FEC compared to DFE (additional advantage)
- This contribution is currently for awareness and any possible action requires further study and work