

Module output BER requirements with inner FEC

Adee Ran, Cisco

Purpose

- In existing PHY types, the specified maximum BER at the optical module output (after the PMA) is $2.4e-4$
 - Provided that the error statistics are sufficiently random that this results in a frame loss ratio of less than **$1.7e-12$** (200G and 400G) or **$3.4e-12$** (800G and 1.6T) – see for example 167.1.1
 - BER= $2.4e-4$ results in exactly these FLR values for uncorrelated errors
 - The MAC-to-MAC link has maximum FLR of $6.2e-11$ due to additional errors on the AUIs
- 802.3dj adopted an inner FEC for optical PMDs at 200 Gb/s per lane (See [patra 3dj 01b 2303](#))
 - The decoder of this code creates correlated errors (see [lu 3df logic 220425](#), [bliss 3df 01a 220517](#))
 - Convolutional interleaving was proposed to handle the correlated errors, but details are still TBD.
- This contribution includes an **analytical calculation** of the required **module output BER** with inner FEC.
 - The inner FEC correction capability and its coding gain are implementation dependent; therefore, the *inner FEC input BER* is not analyzed.

Executive summary

- When inner FEC is not used, the requirement (assuming uncorrelated errors) is $BER < 2.4e-4$.
- With inner FEC, for a given BER, the FLR depends on the level of RS-FEC interleaving.
- To yield the same FLR when inner FEC is used, the module output BER requirements should be:
 - With 2-CW interleaving (200G/400G links without additional interleaving), $BER < 2.1e-5$ (1 order of magnitude lower)
 - With 4-CW interleaving (800G and 1.6T links without additional interleaving, or 200G/400G with additional PMA interleaving), $BER < 8e-5$ (~0.5 order of magnitude lower)
 - With 12-CW interleaving (convolutional interleaver), $BER < 2.85e-4$
- For lower FLR targets, the differences are larger.

RS codeword Interleaving

- The input to the inner FEC is RS-FEC symbol-muxed and interleaved
 - 800G and 1.6T: 4-way interleaved
 - 200G and 400G: 2-way interleaved; there is a proposal to increase the interleaving from 2 to 4
- Convolutional interleaver can increase the interleaving from 4-way to 12-way
 - 8-way and possibly 6-way are also options, but are not analyzed here.
- The latency associated with increased interleaving is a recurring concern, and it has been mentioned that it might be possible to avoid it.

Characteristics of the inner FEC

- The code has a payload of 120 bits = 12 RS-FEC symbols and 8 parity bits; $d_{\min}=4$
 - Soft decoding is assumed
 - The decoder does not mark uncorrectable blocks
- At the decoder output, the number of bit errors on each 128-bit block can be:
 - 0 (usually, when correction succeeds)
 - 4 (occasionally, when correction fails)
 - 6 or 8 (rarely, when correction fails badly)
- The Hamming interleaver de-correlates decoder input samples, so the locations of the output errors within the 128-bit block are assumed to be independent.
- The assumptions above are based on contributions to this task force ([bliss 3df 01a 220517](#), [riani 3dj 01a 2303](#)); decoder implementations may vary.

Inner code failures and measured BER

- Most of the bit errors will contribute to the measured BER (except errors in the parity bits)
- Ignoring the case of errors in the parity bits, the measured BER can be expressed as:

$$BER_{measured} \approx \frac{4 \cdot Prob(4 \text{ errors}) + 6 \cdot Prob(6 \text{ errors}) + 8 \cdot Prob(8 \text{ errors})}{120}$$
$$\approx \frac{Prob(4 \text{ errors})}{30}$$

- The inner FEC failure ratio (IFFR) can be estimated from the measured BER (regardless of interleaving):

$$IFFR = \frac{\text{error blocks}}{\text{total blocks}} \approx 30 \cdot BER_{measured}$$

- **In this analysis I assume all inner FEC failures create 4 errors**
 - 6 and 8 error events are rare and have a small effect on the measured BER
 - Their effect on RS-FEC is larger, and it complicates the analysis
 - The results below are lower bounds (real results would only be worse)

Distribution of RS-FEC symbol errors with 12-way CI (full convolutional interleaver) Each color represents a CW

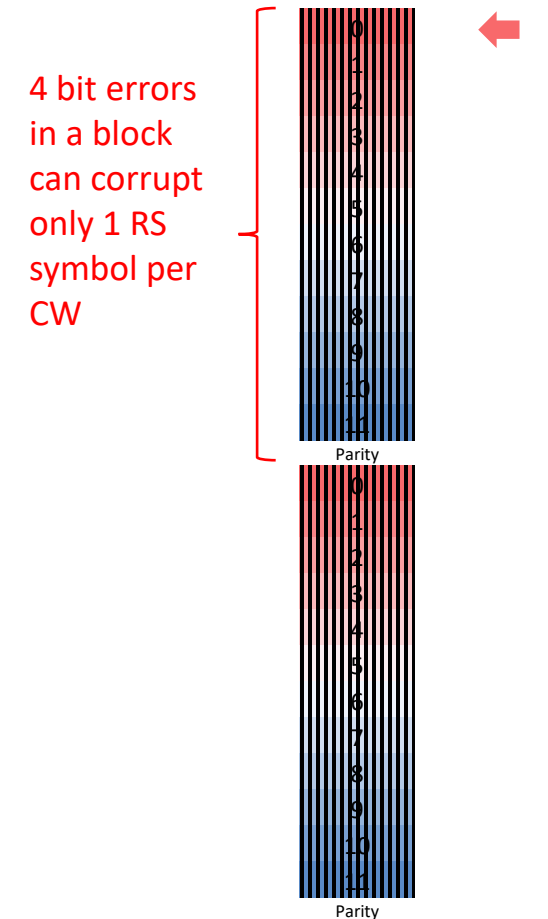
- If full (12-way) CI is used, every inner FEC block is distributed to 12 RS-FEC codewords, and either 0 or 1 RS symbol is affected on each codeword.
- For a given codeword, the probability of having an RS symbol error, given an inner FEC failure (IFF) with 4 bit errors, is

$$P_{CI=12}(1 \text{ error} | IFF) = \left(\frac{118!}{(118 - 4)!} \right) / \left(\frac{128!}{(128 - 4)!} \right) \approx 0.281$$

- Taking into account the IFFR we get:

$$P_{CI=12}(\#errors = 1) = IFFR \cdot P_{CI=12}(1 \text{ error} | IFF) + (1 - IFFR) \cdot 0 \\ \approx 0.281 \cdot 30 \cdot BER \approx 8.4 \cdot BER$$

- This is lower than $SER = 10 \cdot BER$ obtained without inner FEC, so the BER tolerance is better.
- An RS-FEC codeword spans 544 inner FEC blocks.



Distribution of RS-FEC symbol errors with 4-way CI (800G/1.6T)

- With 4-way interleaving, each inner FEC payload contains 3 symbols for each of the 4 RS-FEC codewords, so up to 3 RS symbols can be affected on each codeword.
- The probability distribution of RS symbol errors given an inner FEC failure with 4 bit errors was calculated numerically to be:

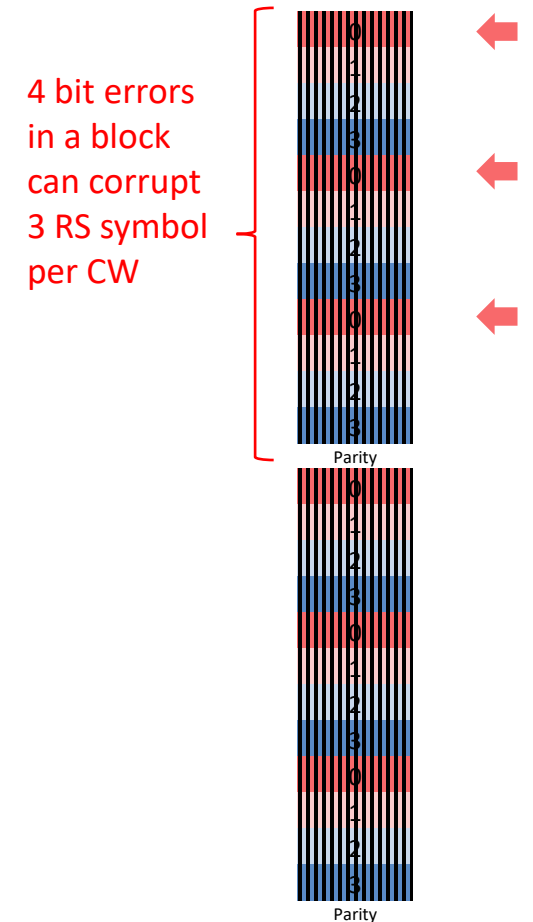
$$P_{CI=4}(\#errors = [0, 1, 2, 3] | IFF) \approx [0.3386, 0.4912, 0.1597, 0.0105]$$

- Taking into account the IFFR we get:

$$P_{CI=4}(\#errors = [0, 1, 2, 3]) \approx IFFR \cdot [0.3386, 0.4912, 0.1597, 0.0105] + (1 - IFFR) \cdot [1, 0, 0, 0]$$

- An RS-FEC codeword spans 544/3 inner FEC blocks; rounding up, 182

Each color represents a CW



Distribution of RS-FEC symbol errors with 2-way CI (200G/400G)

- With 2-way interleaving, each inner FEC payload contains 6 symbols for each of the 2 RS-FEC codewords, so up to 6 RS symbols can be affected on each codeword.
 - For the current analysis, I assume only 4 bit error events, resulting in up to 4 RS symbols.
- The probability distribution of RS symbol errors given an inner FEC failure with 4 bit errors was calculated numerically to be:

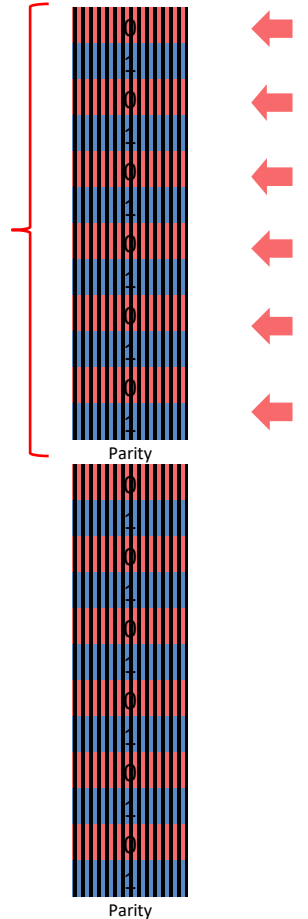
$$P_{CI=2}(\#errors = [0, 1, 2, 3, 4] | IFF) \approx [0.0763, 0.3442, 0.4126, 0.1528, 0.0141]$$
- Taking into account the IFFR we get:

$$P_{CI=2}(\#errors = [0, 1, 2, 3, 4]) \approx IFFR \cdot [0.0763, 0.3442, 0.4126, 0.1528, 0.0141] + (1 - IFFR) \cdot [1, 0, 0, 0, 0]$$
- A codeword spans 544/6 inner FEC blocks; rounding up, 91

Each color represents a CW

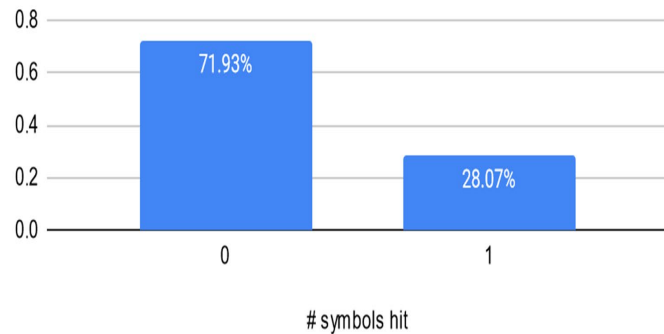
4 bit errors in a block can corrupt 4 RS symbol per CW

But 6 or 8 bit errors can corrupt 6 RS symbols

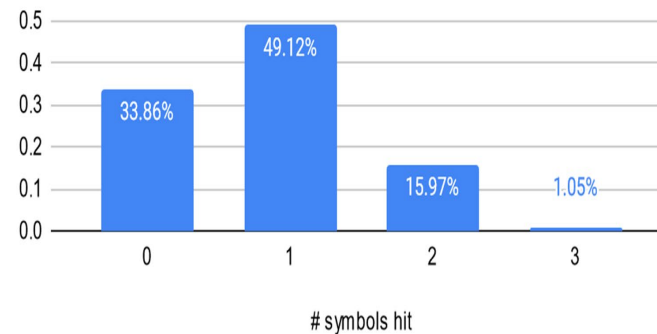


Distribution of RS symbol errors given an inner FEC failure

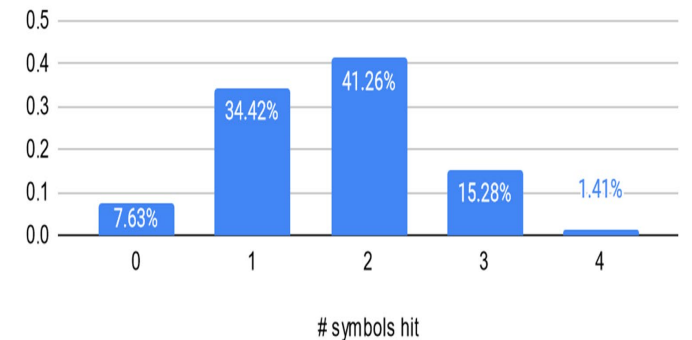
12-way interleaving



4-way interleaving

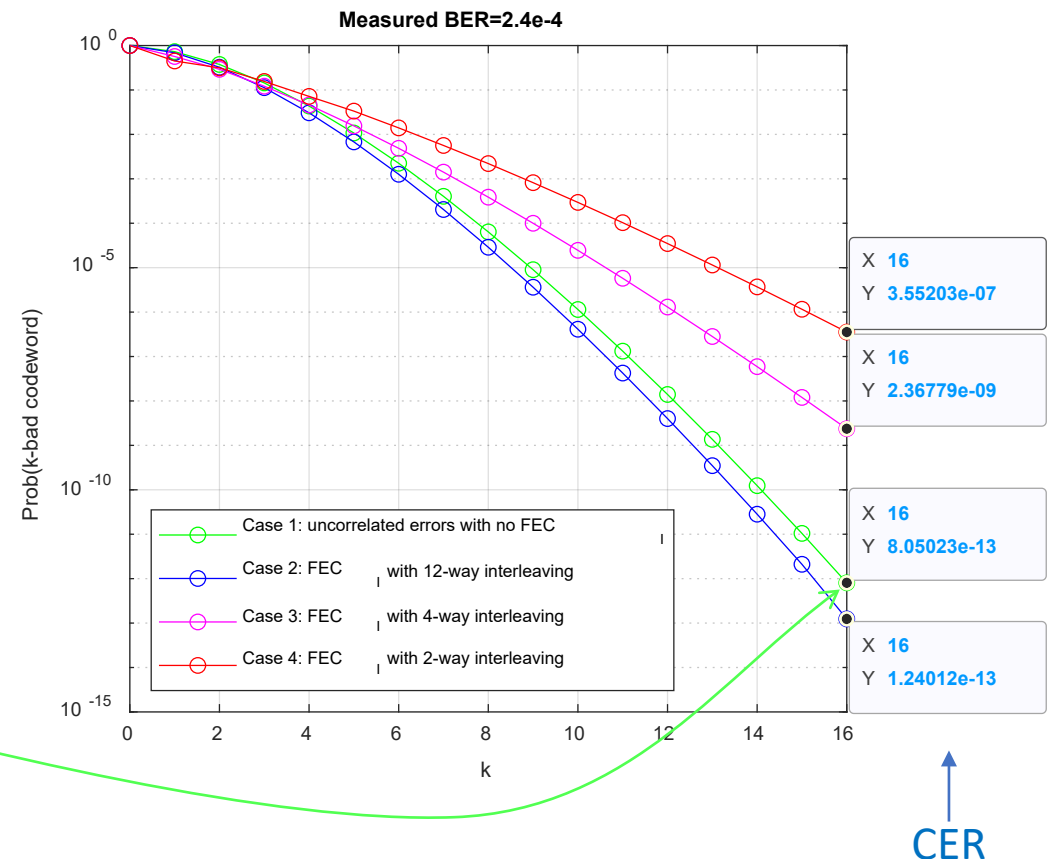


2-way interleaving



RS-FEC CER calculation

- The PDF of number of symbol errors in a full RS codeword can be calculated from $P(\#errors)$ by repeated convolution.
- The complementary cumulative distribution functions (CCDF) with measured BER=2.4e-4 are shown on the right.
 - CCDF at 16 yields the codeword error ratio (CER).
- The FLR allocation for the module-to-module link is:
 - For 200G and 400G: 1.7e-12
 - For 800G and 1.6T: 3.4e-12
 - Both are equivalent to CER < 8e-13
- **With less than 12-way interleaving, BER=2.4e-4 results in much higher CER!**



What measured BER would be acceptable?

- To get CER=8e-13 (equivalent of the FLR target):
 - Without inner FEC: <2.4e-4
 - With inner FEC and 12-way interleaving: **<2.85e-4**
 - With inner FEC and 4-way interleaving (800G/1.6T): **<8e-5**
 - With inner FEC and 2-way interleaving (200G/400G) : **<2.1e-5**
- 200G/400G without additional interleaving is even more sensitive
 - A single IFF event can corrupt 6 symbols; this effect was neglected here
 - If 2-way is used, some guard band should be applied, e.g., <1e-5
 - Increasing interleaving to at least 4-way seems to be preferable!
- For lower FLR target, the penalty of not having full interleaving on required module output BER is larger.

Pre-FEC BER

- ... is not analyzed in this presentation
 - Not easily measurable, and need not be specified
- Due to the FEC coding gain, the effect of interleaving on required pre-FEC BER is likely smaller.
- There is likely a similar small effect on SER in TDECQ/TECQ (not analyzed either).

Summary

- Assuming we preserve the FLR limit of the module-to-module link, the measured BER limit for modules with inner code depends on interleaving level:
 - With 12-way interleaving, the limit should be $\text{BER} < 2.85\text{e-}4$
 - With 4-way interleaving, the limit should be $\text{BER} < 8\text{e-}5$
 - With 2-way interleaving, the limit should be $\text{BER} \ll 2.1\text{e-}5$ (preferably $< 1\text{e-}5$)
- It is recommended to increase the interleaving to 4 for 200G/400G.
- If multiple interleaving levels are allowed, we would need a different measured BER limit for each case.