

# TDECQ Calculation

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IEEE 802.3dj Ad Hoc

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# Review of Cl 180.9.6.4

Let  $x$  be the transmitted symbol,  $x \in \{0,1,2,3\}$

Let  $y$  be the equalized received signal point without added noise

Three thresholds,  $P_{th}(1) < P_{th}(2) < P_{th}(3)$

Four zones, or decision regions

$$\text{Zone}[0] = \{y | y < P_{th}(1)\}$$

$$\text{Zone}[1] = \{y | P_{th}(1) < y < P_{th}(2)\}$$

$$\text{Zone}[2] = \{y | P_{th}(2) < y < P_{th}(3)\}$$

$$\text{Zone}[3] = \{y | y > P_{th}(3)\}$$

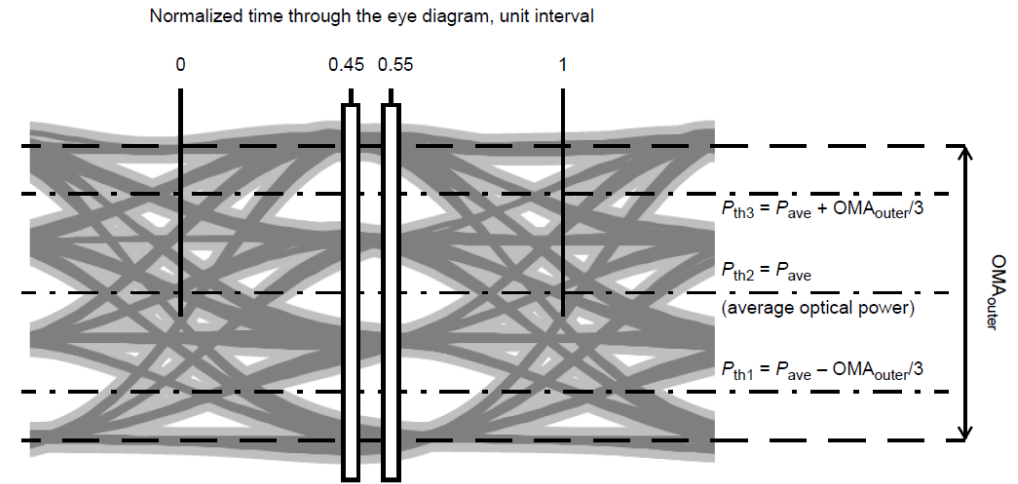


Figure 121-5—Illustration of the TDECQ measurement

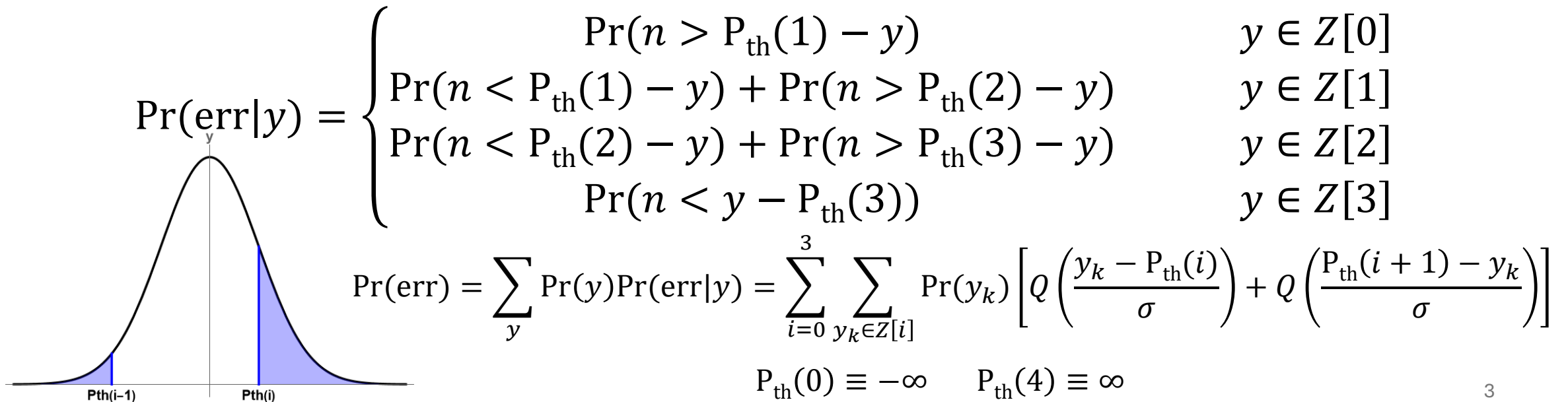
Let  $n$  be the noise added to  $y$  to give  $z$ , i.e.,  $z=y+n$

The decision rule is that the decided symbol based on the noisy  $z$  is

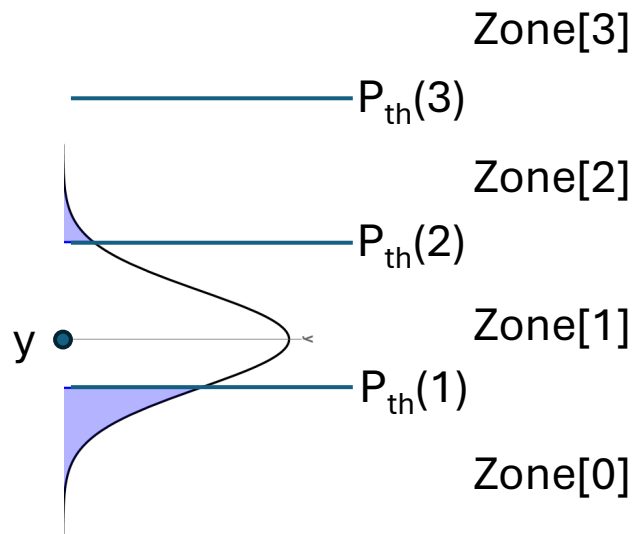
$$\hat{x}(z) = i \text{ where } z \in Z[i]$$

# When is a symbol error made?

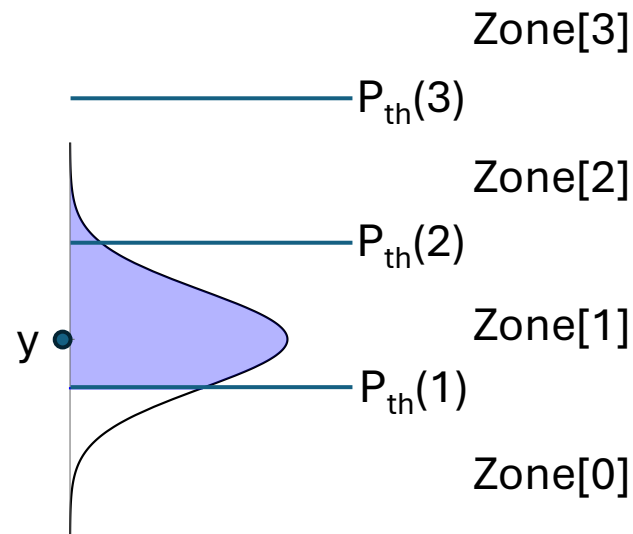
- Suppose  $y$  is in the correct zone to begin with ( $\hat{x}(y) = x$ )
  - This assumes that the eye is open
  - For severe ISI/high transmit noise, this assumption may not be valid
- When the assumption is valid, an error is made if the noise takes  $z$  out of the zone  $y$  is in initially



# Examples



For  $x = 1$  and  $y$  received as shown, probability of error is shaded integral



For  $x = 0$  and  $y$  received as shown, probability of error is shaded integral

When  $y$  starts in the incorrect zone, a symbol error is made if the noise does not put  $z$  back in the correct zone.

- Since the transmitted sequence is known, we should compute the symbol error rate for added noise using the transmitted value and not just assume that it was received in the correct zone.
- This can be approximated using histograms for  $y$  and the appropriate ERFC or Q function.
  - Closed form expressions for Q that give very good approximations can be used

# How CL 180.9.6.4 computes symbol error

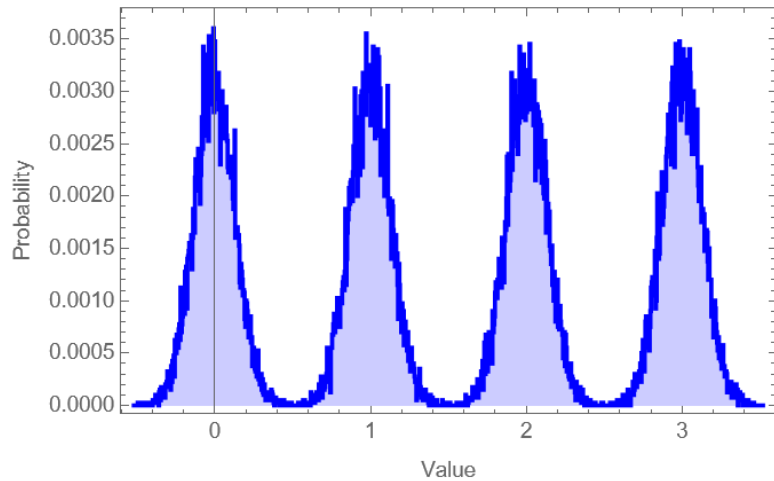
- Clause 180.9.6.4 does not compute symbol error rate as described on the previous slides
  - Instead, for each threshold, it calculates a “Cumulative Distribution Function”

$$\text{CDF}(y_0, P_{\text{th}}(i)) = \begin{cases} \text{Prob}(P_{\text{th}}(i) \leq y \leq y_0), & y_0 > P_{\text{th}}(i) \\ \text{Prob}(y_0 \leq y < P_{\text{th}}(i)), & y_0 < P_{\text{th}}(i) \end{cases}$$

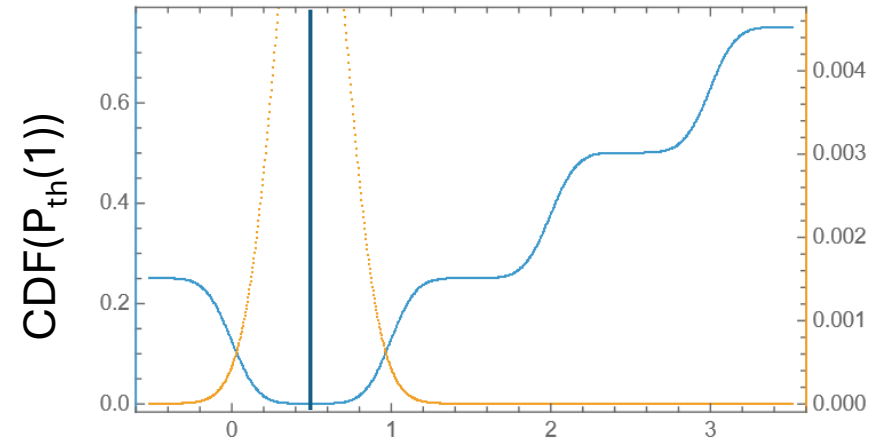
Note: CDF is strictly a misnomer, as a CDF is monotonically non-decreasing from 0 to 1.

- It then computes a weighted sum of the  $\text{CDF}(y_0, P_{\text{th}}(i))$  values over all  $y_0$ , where each value is weighted by the probability that the Gaussian noise is  $P_{\text{th}}(i) - y_0$
- It does this for each of three thresholds, then sums those three probabilities for the final probability of symbol error

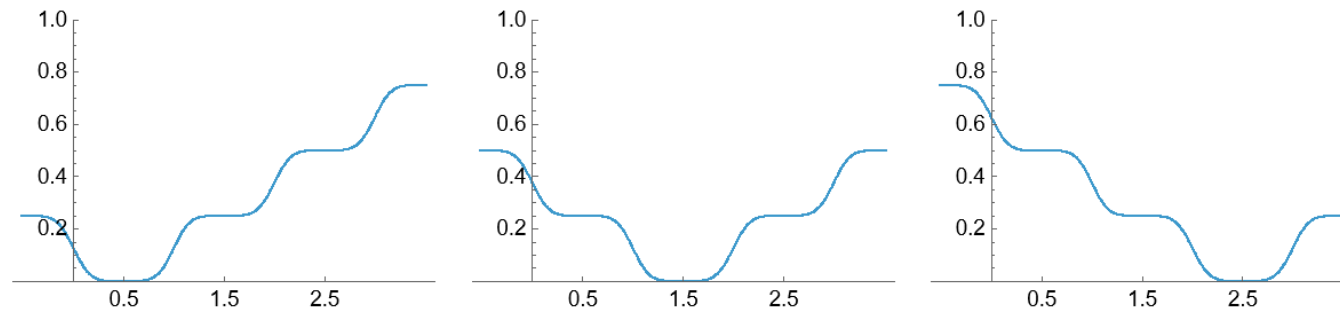
# Example Calculation using CL 180.9.6.4



Distribution of  $y$  about nominal values



CDF and Gaussian Function for Threshold 1



CDFs for All Three Thresholds

The process is repeated and the probabilities of symbol error are summed

# Observations and Question

- The current SER estimation method:
  - Double (or triple) counts some errors by counting the crossing of two thresholds as two symbol errors
  - Does not account for the tail of the Gaussian probability for the extreme  $y$  values, so underestimates that part
  - Underestimates symbol error if  $y$  is in the wrong zone before adding noise
- These issues can be fixed by using the Q-function and using the known transmitted sequence.
- The magnitude of the error has not (yet) been demonstrated.
- Question: Is this worth fixing?

# Thank you

FYI: Contribution “MMSE Calculation of Reference Equalizer Taps for TDECQ” is available showing unconstrained optimization of DFE taps. Optimization subject to tap constraints needs to be added. Contact N. Swenson