# Error Probability and RS-FEC Coding Gain

Ragnar Jonsson – Infineon
October 9, 2025

# Introduction

- The different RS-FEC proposals for IEEE 802.3dm have different error correction capability

- The TDD proposal for 802.3dm [1] uses
  - RS(130,124,8) for the low data rate direction and
  - RS(130,122,8) for the high data rate direction
- The ACT proposal for 802.3dm [2] uses
  - RS(50,46,6) for the low data rate direction
  - RS(360,326,10) for the high data rate direction

- This document evaluates the error correction capability and coding gain for the FEC candidates

[1] https://ieee802.org/3/dm/public/0725/Baseline_Text_for_TDD_PHY_V1.1_07_14_25.pdf
[2] https://ieee802.org/3/dm/public/0925/8023-200_ACT_D0p7a.pdf

# FEC Error Correction

- For RS(N,K,m) RS-FEC the maximum number of correctable errors is

$$n_{\text{correct}} = \frac{N-K}{2}$$

- where
  - *N* is the number of FEC-symbols in each FEC code word
  - *K* is the number of data FEC-symbols in the FEC code word
  - m is the number of bits in each FEC-symbol
- If the number of FEC-symbol errors exceeds $n_{\text{correct}}$, then an uncorrectable FEC error will occur

# PAM-Symbol and FEC-Symbol Error Probability

- The probability of error in each FEC-symbol is*

$$P_E = 1 - (1 - P_e)^{m/b}$$

- where
  - $P_e$ is the probability of error in each PAM-symbol
  - $b$ is the number of bits per PAM-symbol
  - $m/b$ is the number of PAM-symbols per FEC-symbol

** Assuming that there is no PAM-symbol error propagation*

# PAM-Symbol Error Propagation

- PAM-symbol error propagation happens when a feed-back mechanism triggers a follow-up error, for example when Decision Feedback Equalizers (DFE) is used

- In this case the probability of error in each FEC-symbol is

$$P'_E = P_E \cdot \left(1 + \frac{P_p^{(m/b+1)}}{m/b \cdot (1 - P_p)}\right)$$

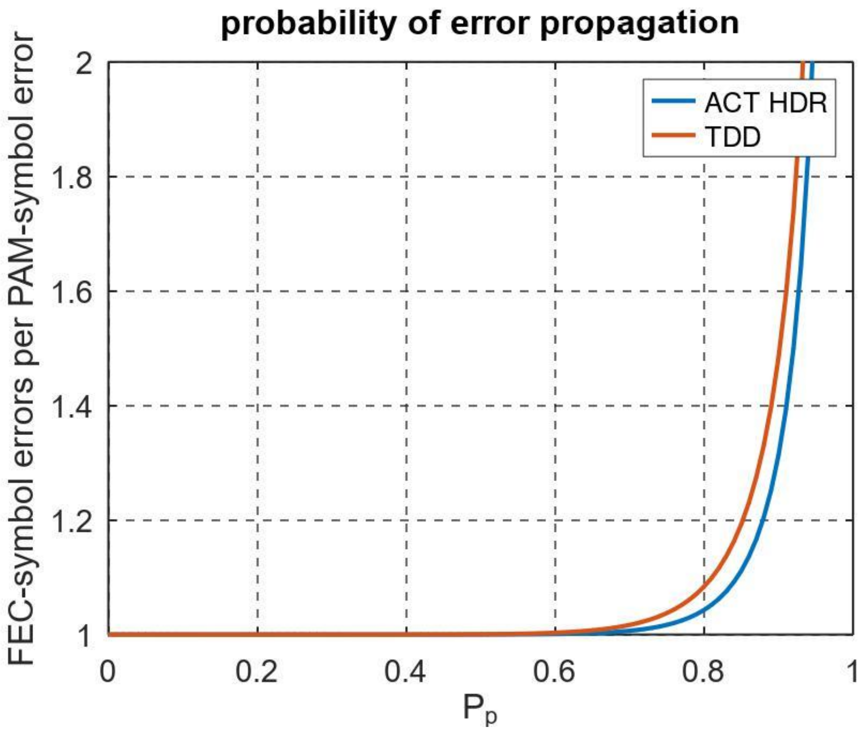
- where
  - $P_p$ is the probability of PAM-symbol error if previous PAM-symbol had error
  - $m/b$ is the number of PAM-symbols per FEC-symbol

# FEC-Symbol Error Dependency on Error Propagation

- The average number of FEC-symbol errors per PAM-symbol error depend on the probability of PAM error propagation, $P_p$

- The system starts to fall apart if $P_p$ is greater than 0.8

- It is reasonable to limit $P_p$ to be no more than 0.8 which means that the average number of FEC errors is less than 1.1 times higher than PAM-symbol errors




NOTE: The difference between the ACT and TDD RS-FEC in the plot is because the ACT uses 10-bit FEC-symbols, while TDD uses 8-bit FEC-symbols

# FEC Error Probability

- The probability of an uncorrectable error can be calculated as

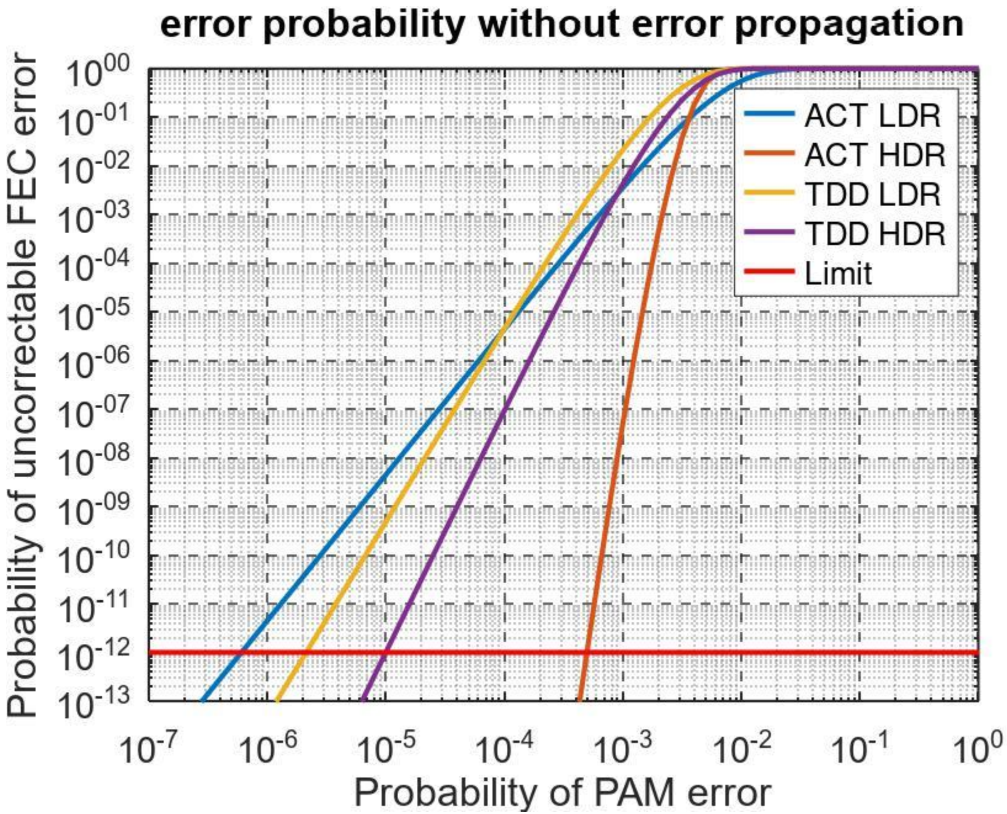
$$P_U = 1 - F_P(n_{correct}, \lambda)$$

- where
  - $F_P(k, \lambda)$ is  cumulative distribution function (CDF) for Poisson distribution
  - $\lambda$ is the average number of errors per FEC code word
  - $n_{correct}$ is the maximum number of FEC-symbol errors that can be corrected

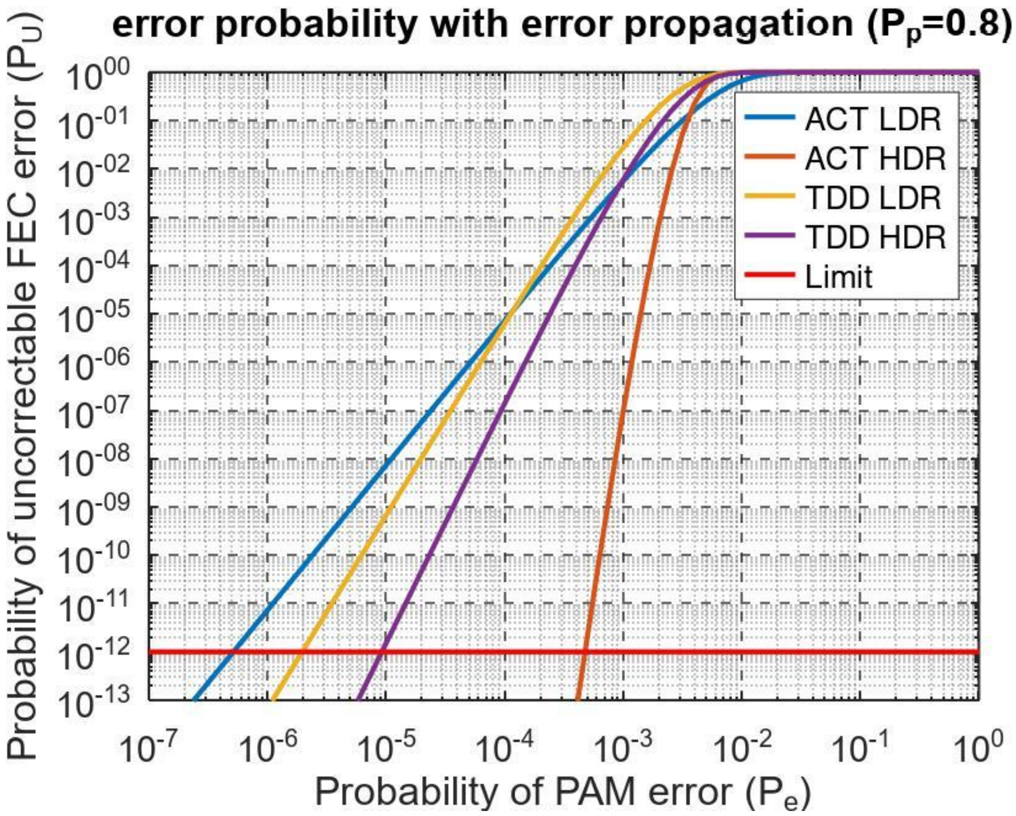
- The average number of errors per FEC code word is

$$\lambda = \mathrm{N} \cdot P_E \cdot \left(1 + \frac{P_p^{(\frac{m}{b}+1)}}{\frac{m}{b} \cdot (1 - P_p)}\right)$$

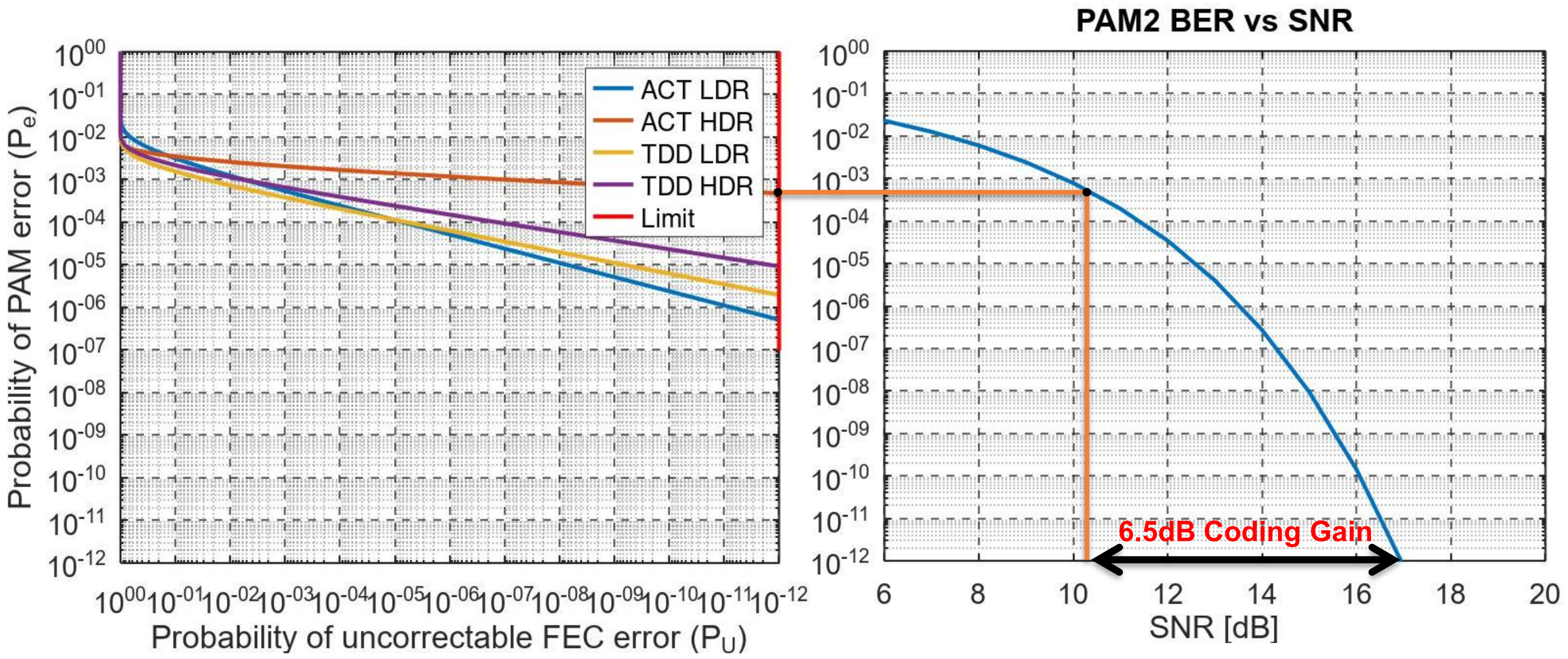
# Probability of Uncorrectable FEC Error (Without Error Propagation)



# Probability of Uncorrectable FEC Error (With Error Propagation)



# FEC Coding Gain




**FEC Coding Gain comes from tolerance to higher PAM-symbol error probability**

# Coding Gain for ACT and TDD FEC Proposals

| | Data Rate [Gbps] | Line Rate [Gbps] | N | K | m | Bits per PAM Symbol | Correctable FEC Errors | Max $P_e$ (NOTE 1) | Min SNR w/o FEC [dB] | Min SNR with FEC [dB] | **Coding Gain [dB]** |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **ACT** | 0.1 US | 0.117 | 50 | 46 | 6 | 1 | 2 | 5.13E-07 | 16.9 | 13.8 | **3.2** |
| | 2.5 | 2.8125 | 360 | 326 | 10 | 1 | 17 | 4.43E-04 | 16.9 | 10.4 | **6.5** |
| | 5 | 5.625 | 360 | 326 | 10 | 1 | 17 | 4.43E-04 | 16.9 | 10.4 | **6.5** |
| | 10 | 11.25 | 360 | 326 | 10 | 2 | 17 | 8.87E-04 | 24.0 | 17.2 | **6.8** |
| **TDD** | 0.1 US | 3 | 130 | 124 | 8 | 1 | 3 | 1.95E-06 | 16.9 | 13.3 | **3.7** |
| | 2.5 | 3 | 130 | 122 | 8 | 1 | 4 | 8.99E-06 | 16.9 | 12.6 | **4.3** |
| | 5 | 6 | 130 | 122 | 8 | 1 | 4 | 8.99E-06 | 16.9 | 12.6 | **4.3** |
| | 10 | 12 | 130 | 122 | 8 | 2 | 4 | 1.80E-05 | 24.0 | 19.5 | **4.5** |

NOTE 1: Max $P_e$ is the maximum PAM-symbol error rate that will not cause more than 10-12 FEC-symbol error rate

**ACT uses bigger RS-FEC and gets more FEC Coding Gain than TDD**

# Summary

- Both ACT and TDD have significant FEC Coding Gain
- FEC does not only give better immunity from impulse noise, but it also gives better immunity from other noise
- When evaluating performance in the presence of noise, including EMI, the evaluation should include FEC Coding Gain

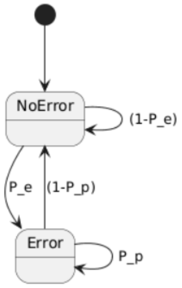
**FEC Coding Gain should not be ignored when evaluating system performance**

# Appendix
# Derivation of Error Probabilities

# Derivation of FEC-Symbol Error with PAM-symbol Error Propagation

## Error Propagation

Error propagation happens when a feed-back mechanism triggers a follow-up error. This is a problem typically associated with Decision Feedback Equalizers (DFE).




The figure above shows a state diagram representing the error propagation. The system starts out in a the NoError state, but can transition to the Error state with probability $P_e$. Otherwise the system stays in the NoError state. Once the system has transitioned into the Error state, it will stay in the Error state with probability $P_p$ (error propagation probability), but otherwise transition to the NoError state.
For functioning PHYs the $P_e$ probability is fairly low, but the error propagation probability, $P_p$, can be high.

## Error Probability

The probability of having error propagation of exactly S PAM-symbols (after the initial error) is

$$P_S = P_p^{S-1}(1 - P_p)$$

The probability of an error propagating beyond the $m$ PAM-symbol FEC-symbol is

$$P_z = \frac{1}{m}\sum_{n=1}^{m} P_p^n = \frac{P_p(1 - P_p^m)}{m(1 - P_p)}$$

The probability of an error propagating over a full $m$ PAM-symbol (one FEC-symbol) is

$$P_m = P_p^m$$

and the probability of the error propagating over k whole FEC-symbol is

$$P_{mk} = P_p^{m \cdot k}$$

The probability of k FEC-symbol errors triggered by single PAM-symbol error is

$$P = P_z \cdot P_p^{m \cdot (k-1)}$$

for $k > 1$.

The probability that any given FEC-symbol has an error is given by the probability that a first PAM-symbol error happens in this FEC-symbol plus the probability that an error propagated into this FEC-symbol from a previous FEC-symbol. This probability is given by

$$P_E' = P_E + P_E \cdot P_z \cdot \sum_{k=1}^{oo} P_p^{m \cdot k} = P_E + \frac{P_E \cdot P_z \cdot P_p^m}{1 - P_p^m} = P_E + \frac{P_E \cdot P_p^m}{1 - P_p^m}\frac{P_p(1 - P_p^m)}{m(1 - P_p)}$$

This simplifies to

$$P_E' = P_E \cdot (1 + \frac{P_p^{(m+1)}}{m \cdot (1 - P_p)}).$$

# Derivation of FEC Error Probability

## FEC Error Probability

Reed-Solomon Forward Error Correction (RS-FEC) can correct limited number of symbol errors. For RS($N, K, m$) RS-FEC the maximum number of correctable errors is

$$n_{correct} = \frac{N-K}{2},$$

where $N$ is the number of FEC symbols in each FEC code word, and $K$ is the number of data FEC symbols in the FEC code word. Each FEC symbol consists of $m$ bits, and there can be more than one incorrect bit whin each wrong FEC symbol, and the RS-FEC can still correct it. If the number of FEC symbol errors exceeds $n_{correct}$, then an uncorrectable FEC error will occur.

If the probability of PAM symbol error is $P_e$, and each PAM symbol represents $b$ bits, then the probability of error in each FEC symbol is

$$P_E = 1 - (1 - P_e)^{m/b},$$

assuming that $P_e$ is small enough that it is unlikely that the same FEC symbol has two independent random errors.
For FEC code words, with N FEC symbols, the average error rate is

$$\lambda = N \cdot P_E = N \cdot (1 - (1 - P_e)^{m/b}),$$

Assuming that the errors are independent and will arrive independent of how long it is since the last error, then the number of errors in each FEC code word will have Poisson distribution, and the probability of $k$ errors is

$$P(k) = \frac{\lambda^k \cdot e^{-\lambda}}{k!}.$$

The probability of having more than $k_0$ FEC symbol errors in any FEC code word is

$$P(k > k_0) = \sum_{k>k_0} \frac{\lambda^k \cdot e{-}\lambda}{k!} = 1 - \sum_{k \le k_0} \frac{\lambda^k \cdot e{-}\lambda}{k!} = 1 - \frac{\Gamma(k_0+1, \lambda)}{k_0!} = 1 - F_P(k_0, \lambda).$$

Where $F_P(k_0, \lambda)$ is the cumulative distribution function (CDF) for Poisson distribution for the value $k_0$ and the mean value $\lambda$. The probability of an uncorrectable error can now be calculated as

$$P_U = 1 - F_P(n_{correct}, \lambda).$$

Part of your life. Part of tomorrow.