# Partial-Response MMSE for MLSD: Targeted Updates to Annex 178A for 400 Gbps

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# Supporters

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# Abstract

This contribution examines targeted changes to Annex 178A to accommodate partial-response equalization for MLSD with precursor and postcursor components.

The existing Annex 178A formulation has much of the required framework.

The intent is a basis for discussion and consensus building in IEEE P802.3dv and seeking feedback on technical sufficiency and suitability for inclusion in the baseline

*MMSE Minimum Mean Squared Error
*MLSD Maximum Likelihood Sequence Detector

# Extend 178A.1.10 MLSD to evaluate the generalized PR response

- Explore partial-response equalization $\alpha_{-1}D^{-1} + 1 + \alpha_1 D^1$
- Replace DFE syntax with  partial response (PR)
- Don't force MMSE to eliminate useful precursor energy.
- Allow MMSE to optimize a precursor + cursor + postcursor PR target.
- This can improve the equalized signal before MLSD is considered.
- MLSD enables the PR solution but is not the source of the MMSE benefit.
- Apply generalized partial-response cursor locations to MMSE in **Annex 178A.1.8.1** *Receiver discrete-time equalizer coefficients.*
  - Only minor changes in syntax required.
- Use Partial Response Equalization Maximum-Likelihood Sequence Detection based on MMSE and **178A.1.10** *Maximum likelihood sequence detection.*
  - Estimated approach with small changes to 178A.1.10

# Overview of proposed Annex 178A edits

- Replace DFE syntax with a generalized partial response (PR) target (precursor + cursor + postcursor) in 178A.1.8.1
- Update the reference receiver block diagram: PR filter, $COM_{DFE}$ becomes $COM_{PR}$
- Generalize equations and indexes in 178A.1.10 (MLSD) and 178A.1.10.2 (sequence noise distribution); add index k for expanded cursors
- Basic MMSE matrix equation unchanged; only minor syntax changes required

# Replace generalize feedback filter in partial response in receiver block diagram

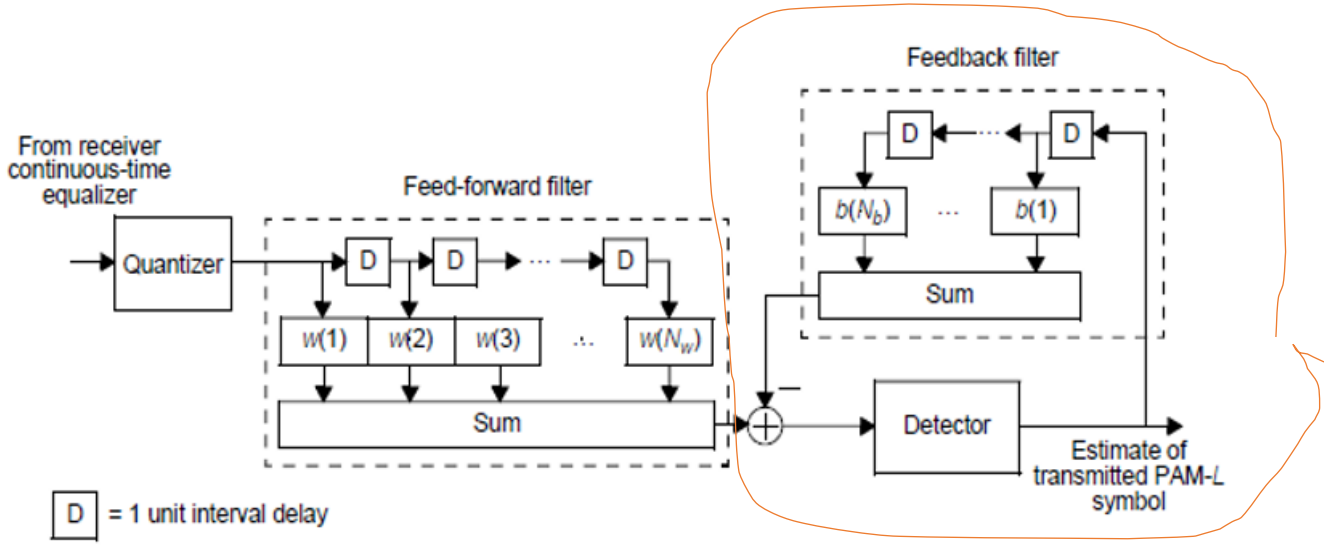
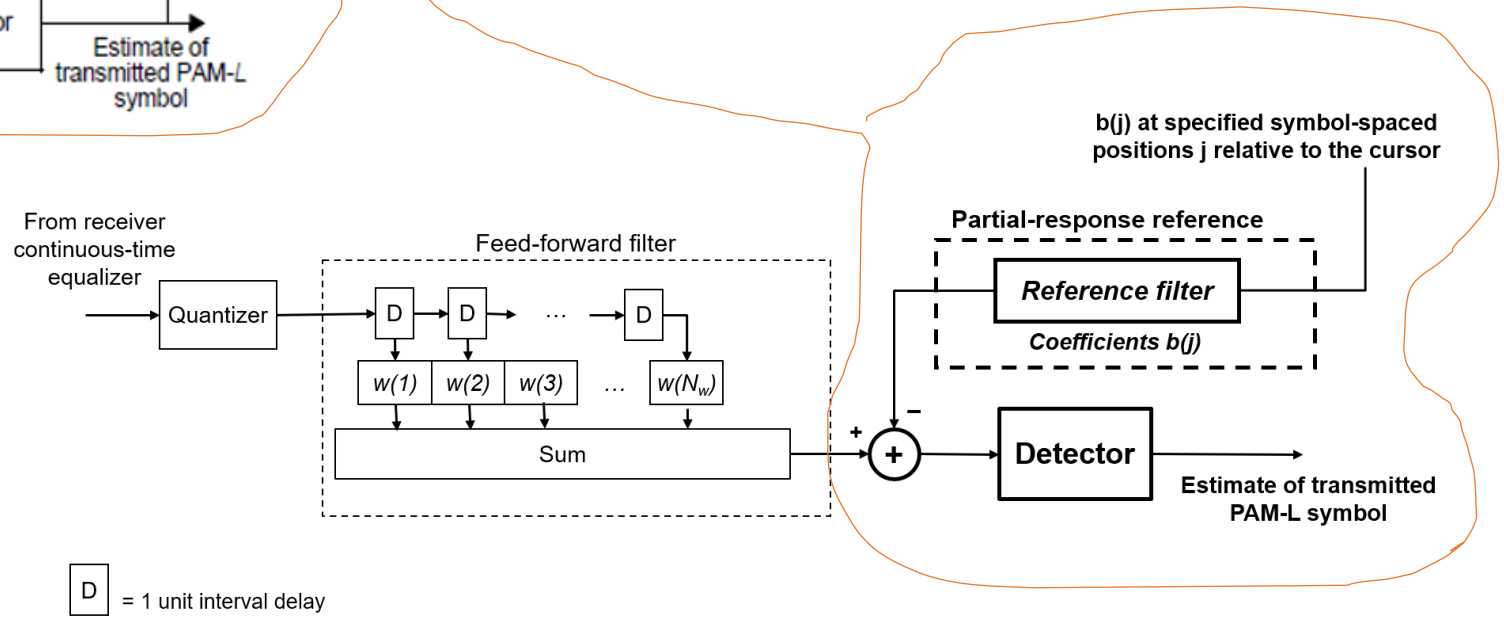



**Figure 178A–10—Receiver discrete-time equalizer**



# Number pre-cursor coefficients, $N_{bn}$, added table

**Table 178A–10—Summary of receiver discrete-time equalizer parameters**

| Parameter | Symbol | Units |
|---|---|---|
| Number of pre-cursor taps | $d_w$ | — |
| Number of fixed-position taps | $N_{fix}$ | — |
| Number of floating tap groups | $N_{wg}$ | — |
| Number of taps per floating tap group | $N_{wf}$ | — |
| Highest allowed tap index | $N_{max}$ | — |
| Normalized upper limit on feed-forward coefficient $w(j)$ | $w_{max}(j)$ | — |
| Normalized lower limit on feed-forward coefficient $w(j)$ | $w_{min}(j)$ | — |
| Number of feedback taps | $N_b$ | — |
| Normalized upper limit on feedback coefficient $b(j)$ | $b_{max}(j)$ | — |
| Normalized lower limit on feedback coefficient $b(j)$ | $b_{min}(j)$ | — |

**Table 178A_10 -- Summary of receiver discrete-time equalizer parameters**

| Parameter | Symbol | Units |
|---|---|---|
| Number of pre-cursor taps | $d_w$ | |
| Number of fixed-position taps | $N_{fix}$ | |
| Number of floating tap groups | $N_{wg}$ | |
| Number of taps per floating tap group | $N_{wf}$ | |
| Highest allowed tap index | $N_{max}$ | |
| Normalized upper limit on feed-forward coefficient $w(j)$, where $j$ is the feed-forward tap index | $w_{max}(j)$ | |
| Normalized lower limit on feed-forward coefficient $w(j)$ where $j$ is the feed-forward tap index | $w_{min}(j)$ | |
| Number of positive partial-response coefficients | $N_b$ | |
| Number of negative partial-response coefficients | $N_{bn}$ | |
| Normalized upper limit on partial-response coefficient $b(j)$, where $j$ is the partial-response coefficient index is relative to the cursor | $b_{max}(j)$ | |
| Normalized lower limit on partial-response coefficient $b(j)$ where $j$ is the partial-response coefficient index is relative to the cursor | $b_{min}(j)$ | |

Negative indexes are included for PR coefficients

# No change to the basic matrix equation

$$\begin{bmatrix} w \\ b \\ \lambda \end{bmatrix} = \begin{bmatrix} R & -H_b^T & -h_0^T \\ -H_b & I_b & z_b^T \\ h_0 & z_b & 0 \end{bmatrix}^{-1} \begin{bmatrix} h_0^T \\ z_b^T \\ 1 \end{bmatrix} \quad (178A–33)$$

where

| | |
|---|---|
| $R$ | is the $N_w \times N_w$ matrix $H^T H + R_{nn}/\sigma_X^2$ and $H^T$ is the matrix transpose of $H$ |
| $h_0$ | is row $d+1$ from $H$ |
| $H_b$ | is the $N_b \times N_w$ matrix formed from rows $d+2$ to $d+N_b+1$ from $H$ |
| $I_b$ | is a $N_b \times N_b$ identity matrix |
| $z_b$ | is a $1 \times N_b$ vector of zeros |

- No change to $b_{lim}$, $w_{lim}$ , $\boldsymbol{\sigma_e^2}$, and COM computation processing

## Refinement of $H_b$, $I_b$, and $z_b$

- $N_b + N_{bn}$ replaces $N_b$ in $H_b$, $I_b$, and $z_b$.

$$\begin{bmatrix} w \\ b \\ \lambda \end{bmatrix} = \begin{bmatrix} R & -H_b^T & -h_0^T \\ -H_b & I_b & z_b^T \\ h_0 & z_b & 0 \end{bmatrix}^{-1} \begin{bmatrix} h_0^T \\ z_b^T \\ 1 \end{bmatrix} \quad (178A\text{-}33)$$

Where:

| Symbol | Definition |
|---|---|
| $R$ | is the $N_w \times N_w$ matrix $H^T H + R_{nn}$, where $H^T$ is the transpose of $H$. |
| $h_0$ | is row $d+1$ of $H$. |
| $H_b$ | is the $(N_b + N_{bn}) \times N_w$ matrix formed from the rows of $H$ at the partial-response coefficient indices $k$, where row $d+1+j$ corresponds to partial-response coefficient $b(j)$. |
| $I_b$ | is the $(N_b + N_{bn}) \times (N_b + N_{bn})$ identity matrix. |
| $z_b$ | is the $1 \times (N_b + N_{bn})$ vector of zeros. |

# Change $COM_{DFE}$ to $COM_{PR}$

Syntax change accommodates partial response (PR)

$$COM = COM_{\mathrm{DFE}} + \underbrace{20\log_{10}\left[\frac{-P_{an}^{-1}(DER_{\mathrm{MLSD}})}{A_s}\right]}_{\boldsymbol{\Delta COM}}$$

$$COM = COM_{\mathrm{PR}} + 20\log_{10}\left[\frac{-P_{an}^{-1}(DER_{\mathrm{MLSD}})}{A_s}\right]$$

# "*178A.1.10 Maximum likelihood sequence detection*" adjusted for partial response

The receiver discrete-time equalizer coefficients are determined using the procedure defined in 178A.1.8.1 using the parameters defined in Table 178A–10 but with the value of $N_b$ set to 1. COM is then computed as defined in 178A.1.9 and the resulting value is labeled $COM_{DFE}$. The value of $COM_{DFE}$ and the feedback filter coefficient $b_{lim}(1)$, along with the corresponding noise and residual intersymbol interference computed at the output of the feed-forward filter, are used to calculate a modification to $COM_{DFE}$ that represents the advantage the MLSD-based receiver has over the DFE-based receiver. This modification is defined by Equation (178A–49).

$$COM = COM_{DFE} + 20\log_{10}(-P_{an}^{-1}(DER_{MLSD})/A_s) \quad (178A–49)$$

where $P_{an}(y)$ is defined in 178A.1.10.1 and $DER_{MLSD}$ is defined by Equation (178A–50).

$$DER_{MLSD} = \sum_{j=1}^{\infty}\left(\frac{L-1}{L}\right)^{j-1} P_j\left(-A_s\frac{(u_j^T u_j)^{3/2}}{(u_j^T V_j u_j)^{1/2}}\right) \quad (178A–50)$$

where

| | |
|---|---|
| $P_j(y)$ | is defined in 178A.1.10.2 |
| $u_j$ | is a $(j+1)\times 1$ vector whose elements are defined by Equation (178A–51) |
| $V_j$ | is defined in 178A.1.10.3 |

$$u_j(i) = \begin{cases} 1 & i = 1 \\ (-1)^{i-1}(1-b_{lim}(1)) & 2 \le i \le j \\ (-1)^i b_{lim}(1) & i = j+1 \end{cases} \quad (178A–51)$$

If the value of COM calculated by Equation (178A–49) is less than $COM_{DFE}$, then the value of COM is set to be equal to $COM_{DFE}$.

The receiver discrete-time equalizer coefficients are determined using the procedure defined in 178A.1.8.1 and the parameters defined in Table 178A-10. Since $b$ is partial response $1 \times (N_b + N_{bn})$ row vector coefficients, $k \in 1\ to\ (N_b + N_{bn})$;

COM is then computed as defined in 178 A.1.9 and the resulting value is labeled $COM_{PR}$. The value of $COM_{PR}$ and the partial response filter coefficient $b_{lim}(k)$, along with the corresponding noise and residual inter symbol interference computed at the output of the feed-forward filter, are used to calculate a modification to $COM_{PR}$ that represents the advantage the MLSD-based receiver has over the PR-based receiver. This modification is defined by Equation (178A-49).

$$COM = COM_{PR} + 20\log_{10}\left[\frac{-P_{an}^{-1}(DER_{MLSD})}{A_s}\right] \quad (178A-49)$$

where $P_{an}$ is defined in 178A.1.10.1 and $DER_{MLSD}$ is defined by Equation (178A-50).

$$DER_{MLSD} = \sum_{k=1}^{N_{bn}+N_b}\sum_{j}^{\infty}\left(\frac{L-1}{L}\right)^{j-1} P_{j,k}\left(-A_s\frac{\left(u_{j,k}^T u_{j,k}\right)^{3/2}}{\left(u_{j,k}^T V_j u_{j,k}\right)^{1/2}}\right) \quad (178A-50)$$

Where:

$P_{j,k}$ is defined in 178A.1.10.2

$u_{j,k}$ is a $(j+1)\times k$ vector whose elements are defined by Equation (178A-51)

$V_j$ is defined in 178A.1.10.3

$$u_{j,k}(i) = \begin{cases} 1 & i = 1 \\ (-1)^{i-1}(1-b_{lim}(k)) & 2 \le i \le j \\ (-1)^j b_{lim}(k) & i = j+1 \end{cases} \quad (179A-51)$$

If the value of COM calculated by Equation (178A-49) is less than $COM_{PR}$, then the value of COM is set to be equal to $COM_{PR}$.

# Closer look: Index *k* added for expanded partial response cursors

where $P_{an}$ is defined in 178A.1.10.1 and $DER_{MLSD}$ is defined by Equation (178A-50).

$$DER_{\mathrm{MLSD}} = \sum_{k=1}^{N_{bn}+N_b} \sum_{j}^{\infty} \left(\frac{L-1}{L}\right)^{j-1} P_{j,k} \left( -A_s \frac{\left(u_{j,k}^T u_{j,k}\right)^{3/2}}{\left(u_{j,k}^T V_j u_{j,k}\right)^{1/2}} \right)$$

Where:

$P_{j,k}$ is defined in 178A.1.10.2

$u_{j,k}$ is a (j + 1) × k vector whose elements are defined by Equation (178A-51)

$V_j$ is defined in 178A.1.10.3

$$u_{j,k}(i) = \begin{cases} 1 & i = 1 \\ (-1)^{i-1}(1 - b_{\mathrm{lim}}(k)) & 2 \le i \le j \\ (-1)^{j} b_{lim}(k) & i = j + 1 \end{cases}$$

If the value of COM calculated by Equation (178A-49) is less than $\mathrm{COM_{PR}}$, then the value of COM is set to be equal to $\mathrm{COM_{PR}}$.

- Index $k$ added for expanded partial response cursors
- $k = N_b + N_{bn}$ and is positive because of the way $N_b$ in $H_b$, $I_b$, and $z_b$ *are determined*
- An extra sum is added to account for summing DERs as an estimate for the added cursors

# "*178A.1.10.2 Sequence noise distribution function* augmented for partial responses

**178A.1.10.2 Sequence noise distribution function**

The probability distribution function of the noise corresponding to a sequence of length $j$ is denoted $p_j(y)$. It is initialized by the assignment defined in Equation (178A–54).

$$p_j(y) = \text{conv}\left[p_{an}(y), \frac{1}{|b_{lim}(1)|} p_{an}\left(\frac{y}{b_{lim}(1)}\right)\right] \quad (178A–54)$$

where $p_{an}(y)$ defined in 178A.1.10.1 and conv[$u$, $v$] denotes the convolution of $u$ and $v$.

It is iteratively updated by repeating the assignment defined by Equation (178A–55) $j - 1$ times.

$$p_j(y) = \text{conv}\left[p_j(y), \frac{1}{|1 - b_{lim}(1)|} p_{an}\left(\frac{y}{1 - b_{lim}(1)}\right)\right]$$

The cumulative distribution function of the noise corresponding to a sequence of leng computed using Equation (178A–40) substituting $p_j(y)$ for $p(y)$.

**Subscript $k$ added**

$$b_{lim}(1) \rightarrow b_{lim}(k)$$

**178A.1.10.2 Sequence noise distribution function**

The probability distribution function of the noise corresponding to a sequence of length j is denoted $p_{j,k}(y)$. It is initialized by the assignment defined in Equation (178A-54).

$$p_{j,k}(y) = \text{conv}\left[p_{an}, (y) \frac{1}{|b_{\lim}(k)|} p_{an}\left(\frac{y}{b_{\lim}(k)}\right)\right] \quad (178A-54)$$

where $p_{an}(y)$ defined in 178A.1.10.1 and conv[u, v] denotes the convolution of u and v.

It is iteratively updated by repeating the assignment defined by Equation (178A-55) ($j$-1) times for each $k$.

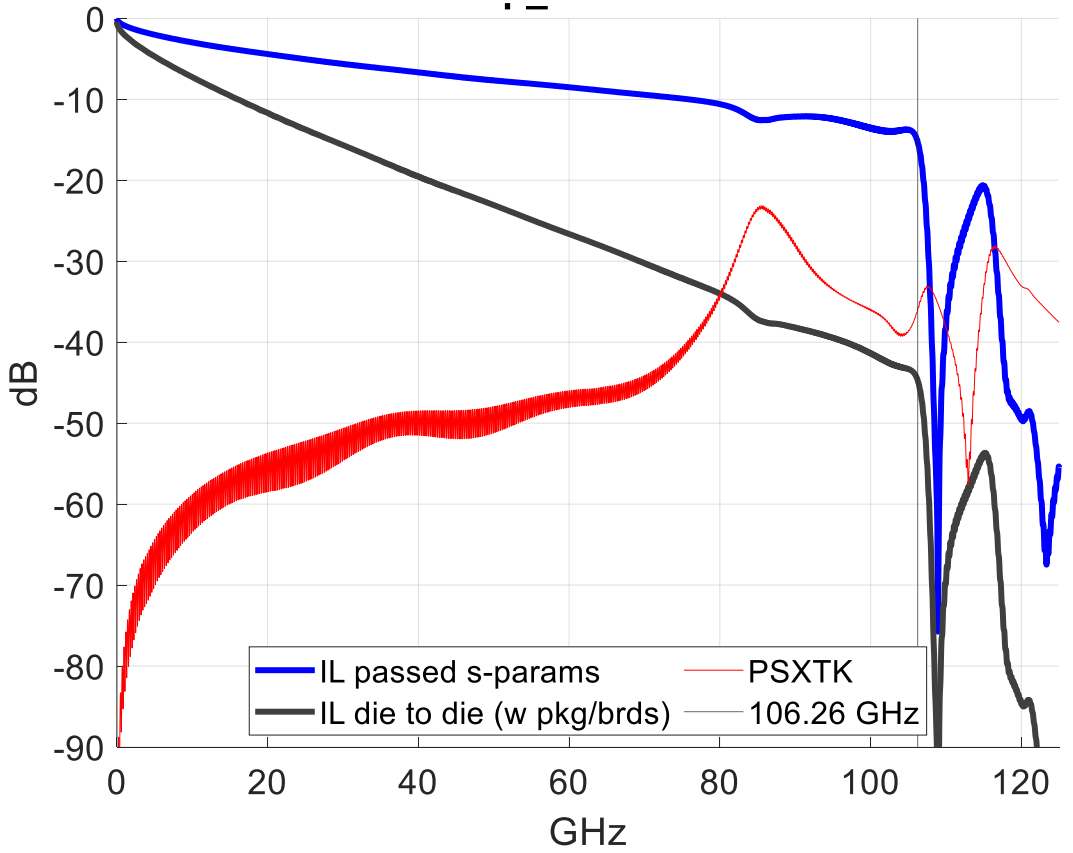
$$p_{j,k}(y) = \text{conv}\left[p_{j,k}, \frac{1}{|1-b_{\lim}(k)|} p_{an}\left(\frac{y}{1-b_{\lim}(k)}\right)\right] \quad (178A_55)$$

The cumulative distribution function of the noise corresponding to a sequence of length $j$, $p_{j,k}(y)$, is then computed using Equation (178A-40) substituting $p_{j,k}(y)$ for $p(y)$.

# PR{1,1} improved COM from −0.8 dB to 5.5 dB on this 44 dB loss channel.

2 test cases

1. COM = −0.8 dB
   DFE1/FFE{8,6} float 2x4 case
   Equivalent syntax PR{0,1}/FFE{8,6}
2. COM = 5.5 dB
   PR{1,1}/FFE{8,6} float 2x4 case
   $\Delta COM$ was 0.9 dB




This CPC to CPC channel has an IL of 44 dB at 106.25GHz with Tx/Rx 45 mm Packages (B).

gore_e4ai_02a_250529\C2C_CPC_EBW_l00mm_thru.s4p

COM test code is in branch MLSD_HO

# COM configuration sheet and sweep table

Values and settings in the spread sheet may not be align with a standard. Please check before using

| Table 93A-1 parameters | | | |
|---|---|---|---|
| **Parameter** | **Setting** | **Units** | **Information** |
| data rate | 425 | Gbit | |
| L | 4 | levels | |
| f_b | 212.50 | #VALUE! | #NUM! |
| f_min | 0.05 | GHz | |
| Delta_f | 0.01 | GHz | |
| R_0 | 46.25 | s | |
| PKG_NAME | PKG_HiR_CLASSB PKG_HiR_CLASSB | | TX RX |
| z_p select | 3 | | |
| M | 32 | | |
| C_d | [0.4e-4 0.9e-4 1.1e-4 ;0.4e-4 0.9e-4 1.1e-4 ]/1.4 | nF | [TX RX] |
| L_s | [0.13 0.15 0.14; 0.13 0.15 0.14 ]/1.4 | nH | [TX RX] |
| C_b | [0.3e-4 0.3e-4 ]/1.4 | nF | [TX RX] |
| **filter and Eq** | | | |
| f_r | 0.55 | *fb | 116.88GHz |
| c(0) | 0.55 | | [min:step:max] |
| c(-1) | [ -0.34:0.02:0 ] | [ -0.34:0.02:0 ] | see if 0 is OK |
| c(-2) | [ 0:0.02:0.14 ] | [ 0:0.02:0.14 ] | see if 0 is OK |
| c(1) | [ -0.2:0.02:0 ] | [ -0.2:0.02:0 ] | see if 0 is OK |
| N_b | 1 | UI | |
| N_bn | 0 | 1 | 2 |
| bmax | [0.85] | [.25 0.85] | [.8 .9 .85 .5 ] |
| bmin | [0] | [ 0 0 ] | [ 0 0 0 0 ] |
| | | | |
| g_DC | [-20:1-0] | [-20:1-0] | |
| f_z | 85.00 | GHz | |
| f_p1 | 85.00 | GHz | |
| f_p2 | 212.50 | GHz | |
| g_DC_HP | [-6:1:-0] | | see if 0 is OK |
| f_HP_PZ | 1.328125 | GHz | |

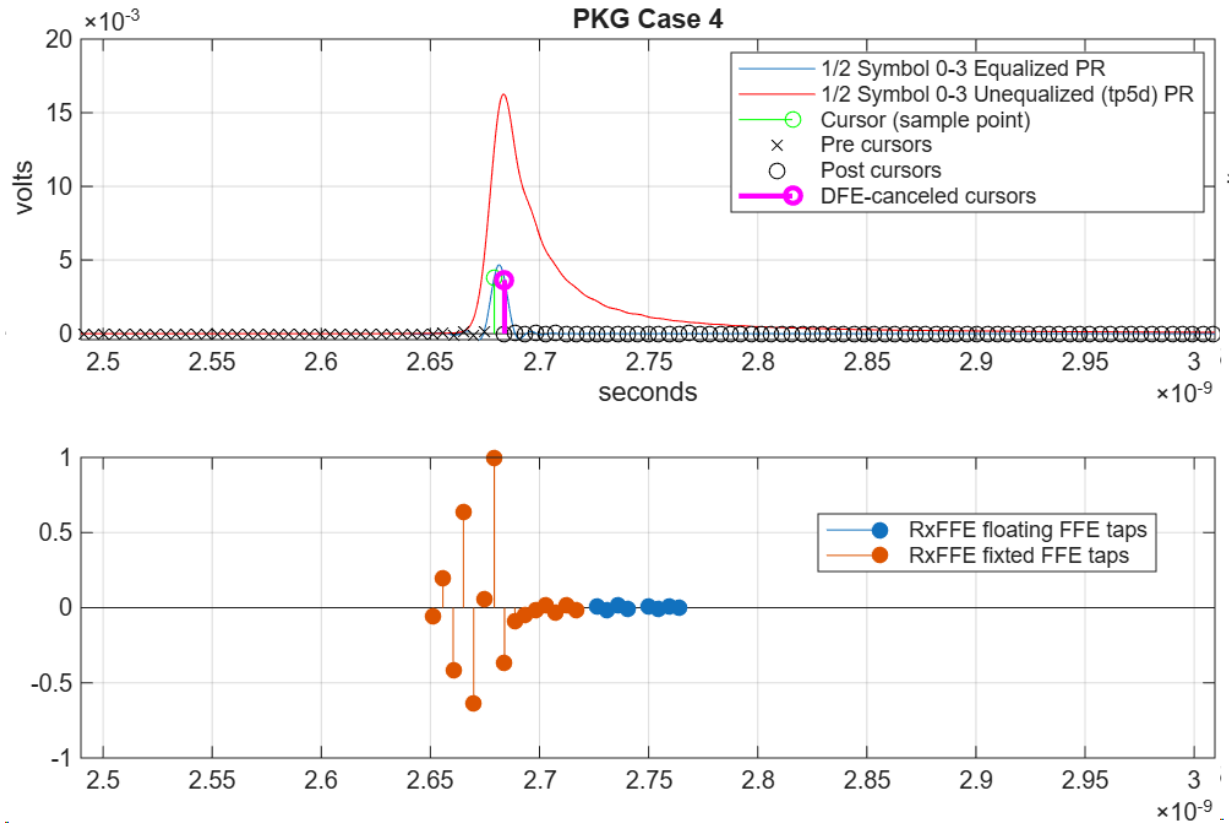
| I/O control | | |
|---|---|---|
| DIAGNOSTICS | 1 | logical |
| DISPLAY_WINDOW | 1 | logical |
| CSV_REPORT | 0 | logical |
| RESULT_DIR | .\results\KRCR_448G_{date}\ | |
| SAVE_FIGURES | 0 | logical |
| Port Order | [ 1 3 2 4 ] | |
| RUNTAG | Test_448C2M_pam4_200G_stop_ | |
| COM_CONTRIBUTION | 0 | logical |
| BREAD_CRUMBS | 1 | logical |
| no_com | 0 | logical |
| **TDR and ERL options** | | |
| TDR | 0 | logical |
| ERL | 0 | logical |
| ERL_ONLY | 0 | logical |
| TR_TDR | 0.01 | ns |
| N | 1600 | logical |
| TDR_Butterworth | 1 | |
| beta_x | 0 | |
| rho_x | 0.618 | |
| TDR_W_TXPKG | 0 | UI |
| N_bx | 0 | |
| fixture delay time | [ 0 0 ] | |
| Tukey_Window | 1 | |
| Z_t | 46.25 | ohm |
| **Noise, jitter** | | UI |
| sigma_RJ | 0.01 | UI |
| A_DD | 0.02 | V^2/GHz |
| eta_0 | 7.00E-09 | dB |
| SNR_TX | 33.5 | |
| R_LM | 0.95 | |
| Txpskew | 0 | |
| Txnskew | 0 | |
| Rxpskew | 0 | |
| Rxnskew | 0 | |

| Floating Tap Control | | | |
|---|---|---|---|
| N_bg | 2 | 0 1 2 or 3 groups | |
| N_bf | 4 | taps per group | |
| N_f | 80 | UI span for floating taps | |
| bmaxg | 0.05 | max DFE value for floating taps | |
| B_float_RSS_MAX | 0.1 | rss tail tap limit (not used) | |
| N_tail_start | 9 | (UI) start of tail taps limit | |
| **Filter: Rx FFE** | | | |
| ffe_pre_tap_len | 6 | UI | |
| ffe_post_tap_len | 8 | UI | |
| ffe_main_cursor_min | 1 | | |
| ffe_pre_tap1_max | 0.7 | rss tail tap limit (not used) | |
| ffe_post_tap1_max | 0.7 | | |
| ffe_tapn_max | 0.7 | | |
| N_qb | 0 | like enob | |
| P_qc | 1E-07 | | |
| **Operational** | | | |
| ERL Pass threshold | 7.3 | dB | |
| COM Pass threshold | 3 | db | |
| DER_0 | 2.00E-04 | | |
| T_r | 0.00200 | ns | |
| FORCE_TR | 1 | logical | |
| PMD_type | C2C | | |
| EW | 0 | | |
| MLSD | 1 | logical | |
| ts_anchor | 1 | | |
| sample_adjustment | [-24 24] | | |
| num_ui_RXFF_noise | 2000 | need enough UI to capture pulse delay dela | |
| | | | |
| zero_pad | 1 | recommended | only for |
| flim | 200.0E+9 | Hz | only for |
| zero_pad_tukey_window_in_fb | 0.01 | fb | only for |
| Impulse response truncation threshold | 0.001 | may speed up computation at the cost of r | |
| Local Search | 2 | use to get a handle on where the ts and ctl | |
| TS_SRCH_MODE | middle | not used if local search = 0 | |

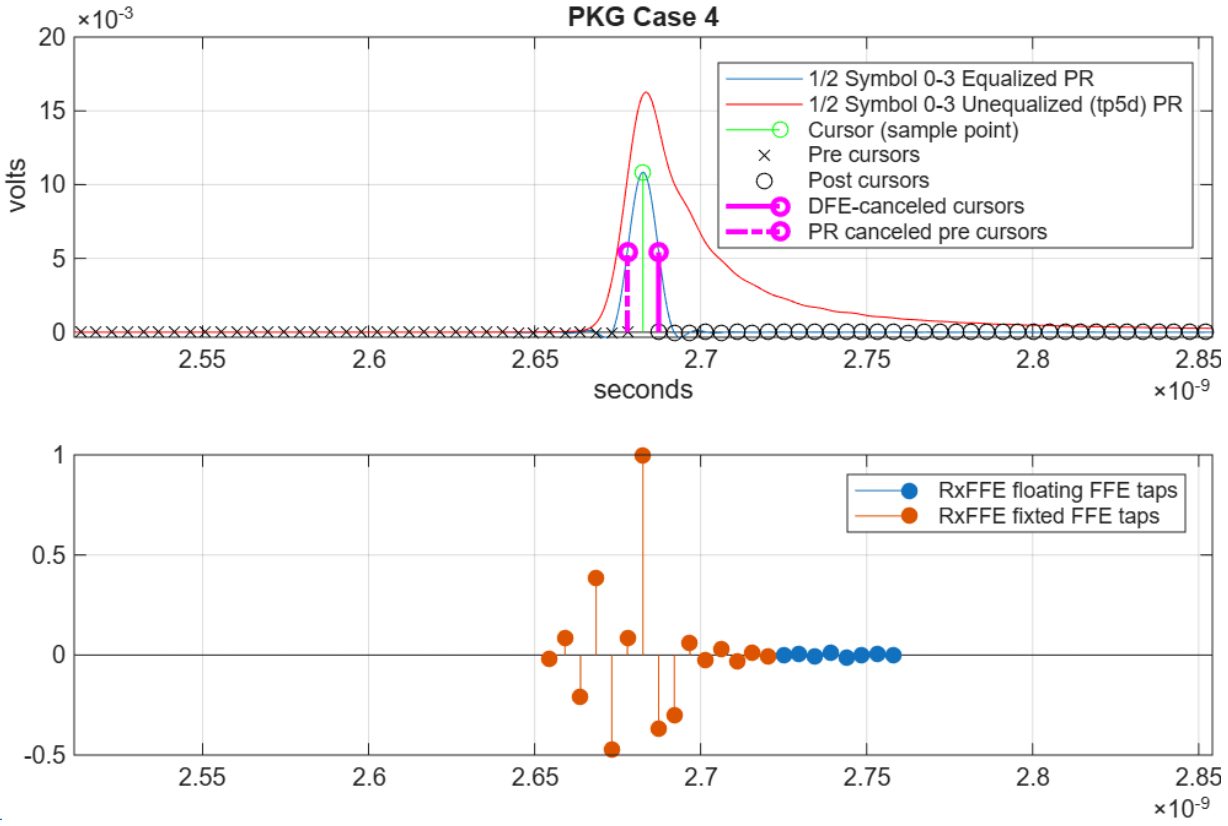
| CASE | N_b | N_bn | bmax | bmin | N_q |
|---|---|---|---|---|---|
| DFE | 1 | 0 | [ 0.85 ] | [ 0 ] | 7 |
| PR{1,1} | 1 | 1 | [0.5 0.5] | [ 0 0 ] | 7 |

# Partial response PR{1,1} equalized pulse response is larger than the DFE case

**DF1/FFE{8,6} w float 2x4**
**$b_{max}$ = 0.85**




**PR{1,1}/FFE{8,6} w float 2x4**
**$b_{max}$= [ 0.5 0.5 ]**



# Summary

- Results show improved COM for partial-response equalization $\alpha_{-1}D^{-1} + 1 + \alpha_1 D^1$
- Path from Annex 178A demonstrated
- Next steps
  - review
  - refinement
  - consensus

✓ COM test code is in branch MLSD_HO

# Thank You!