# Constraint-Coded Modulation for PAM6

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# Agenda

- Introduction
- Bandwidth restriction / limitation for 400 Gbps
  - E2E electrical channel IL and available BW
- Constraint-Coded Modulation concept
  - Overall encoding and decoding scheme
- PAM6 Native excess constellation capacity
- PAM6 Constellation Mapping for mod-2 constraint
  - asymptotic analysis
  - performance simulation
- PAM6 Constellation Mapping for higher moduli constraint (mod-3, 4)
  - asymptotic analysis
  - performance simulation
- PAM6 Hybrid Native-Excess and Supplemental-Resource Coding
  - constellation Mapping
  - performance simulation
- Conventional Hamming-code reference for PAM6 (interleaving disabled)
- Complexity, summary, and conclusions

# Introduction

- PAM6 modulation is being considered for electrical interfaces defined by the project.
- Compared with PAM4 at the same peak amplitude, PAM6 has a smaller minimum decision distance and therefore requires a higher SNR to achieve the same error ratio.
- To support the target channel loss and reach, the probability of correct sequence detection must be improved.
- Increasing the signaling rate would extend the required channel bandwidth and introduce additional channel and receiver penalties.
- This contribution therefore investigates constraint-based PAM6 encoding that uses native excess constellation capacity and joint trellis-based detection.

# PAM6 Is Already Close to the Channel-Bandwidth Limit

## Practical channel bandwidth

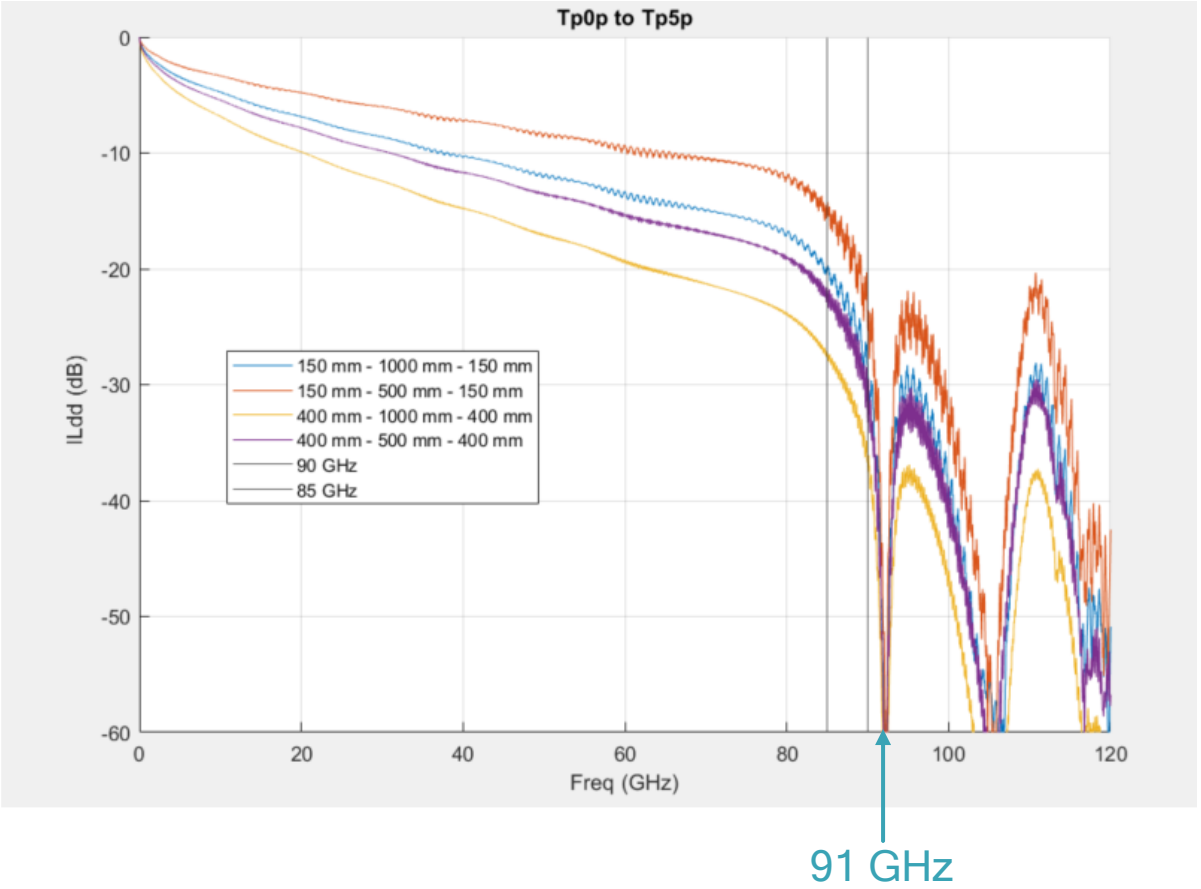
For an approximately 425 Gb/s gross lane rate:

$$R_s = \frac{425\ \text{Gb/s}}{2.5\ \text{bits/symbol}} = 170\ \text{GBd}$$

$$B_N = \frac{R_s}{2} = 85\ \text{GHz}$$

- Uncoded PAM6 requires 85 GHz
- Apparent channel bandwidth is approximately 88 GHz
- A strong high-frequency loss transition occurs above this region
- A potential notch may occur near 91 GHz

**The uncoded PAM6 operating point is already close to the apparent channel-bandwidth limit.**




- 85 GHz: Uncoded PAM6
- Approximately 88 GHz: Apparent channel bandwidth
- Approximately 91 GHz: Potential notch

[1] Source: R. Mellitz et al., “400 GBPL Electrical Objective Kaleidoscope: Reach, Loss, Cutoff, Noise, and COM,” IEEE 802.3 400 Gb/s/Lane Study Group, July 2026

# The Potential Bandwidth Extension Is Only Approximately 3.5%

## Practical channel bandwidth

Uncoded PAM6:

$B_0 = 85\ \text{GHz}$

Provisional practical maximum:

$B_{\max} = 88\ \text{GHz}$

Potential extension:

$Delta\ B = B_{\max} - B_0 = 88 - 85 = 3\ \text{GHz}$

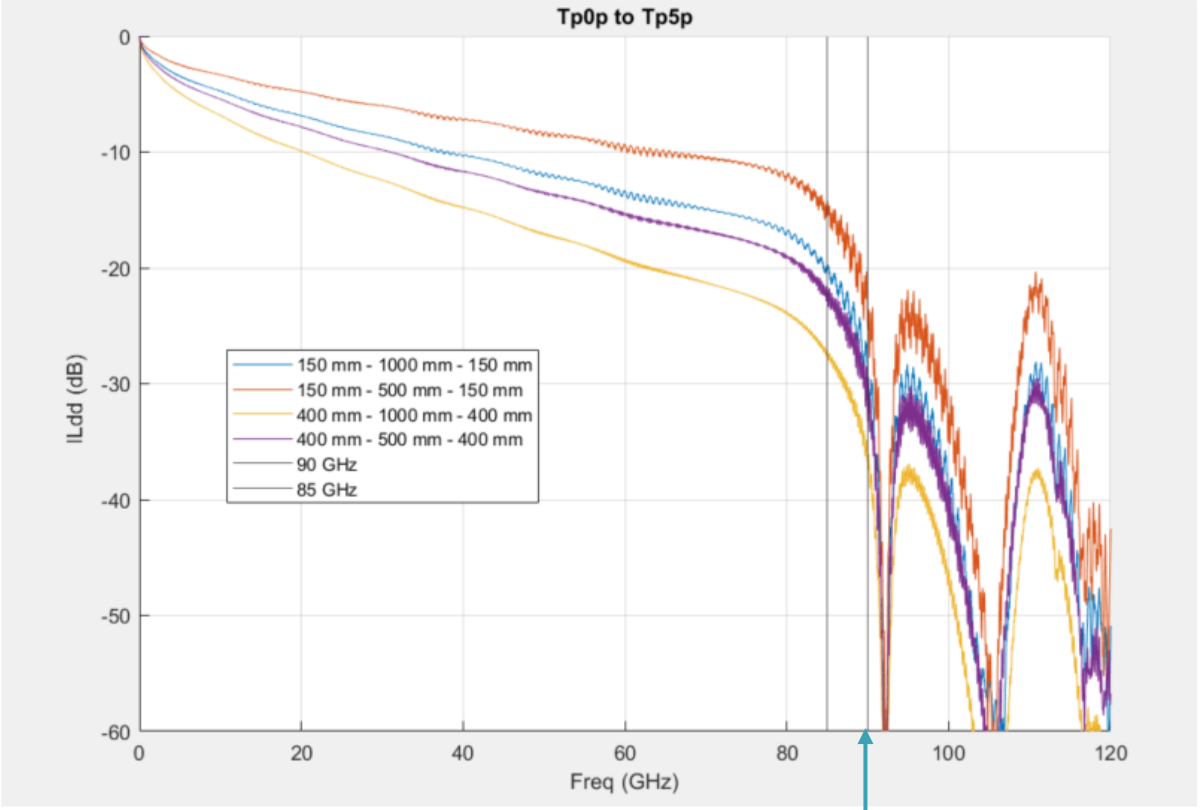
Relative extension:

$$\frac{Delta\ B}{B_0} = \frac{3}{85} = approx\ 3.5\%$$

### The extended sub-band has an additional cost

- Greater channel droop
- Greater equalizer boost
- Increased receiver-noise enhancement
- Less margin to the high-frequency loss transition
- Greater sensitivity to channel variation

**Only approximately 3.5% nominal bandwidth extension may be available, and that extension is not free.**




### Example: Hamming (128,120)

Required signaling-rate factor at equal payload throughput:

$$\frac{128}{120} = 1.0667$$

Required signaling-rate increase: $\frac{Delta_R}{R} = \frac{128}{120} - 1 = 6.67\%$

Required Nyquist bandwidth: $B_H = 85 \cdot \frac{128}{120} \approx 90.7\ \text{GHz}$

# Proposed Constraint-Coded Modulation

Transmitter:

- Generalized constellation mapping
  - Maps payload bits into a PAM6 symbol block
  - Makes full use of the available PAM6 block capacity
  - Supports different block lengths and constraint moduli
  - Is not claimed to provide distance-optimal labeling
- Generalized weighted modulo constraint
  - Restricts the legal PAM6 sequence set
  - Rejects selected low-distance error events
  - Supports modulo-$S$ constraints

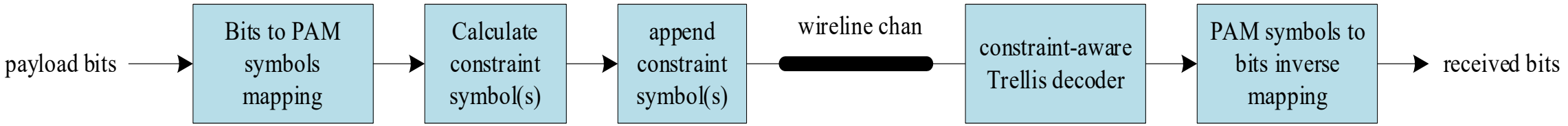
Receiver: Constraint-aware MLSD

- Jointly processes channel memory and constraint state
- Reuses PAM6 Euclidean candidate metrics
- Does not require a separate soft inner decoder

**Two operating modes are evaluated**

- **Native-capacity-only mode:** no additional symbols and no baud-rate increase
- **Hybrid mode:** native PAM6 excess capacity plus a small number of supplemental symbols for higher coding gain

**The proposed method spends native PAM6 constellation capacity and receiver state complexity instead of additional transmission bandwidth.**

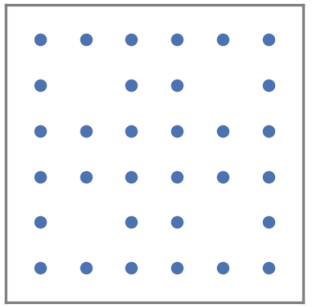


# PAM6 Has Native Excess Constellation Capacity

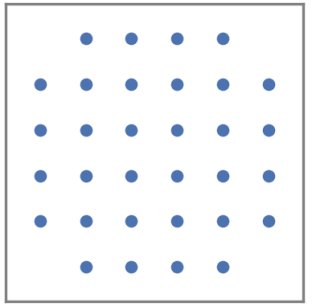
## Conventional PAM6 mapping and excess capacity

- A conventional rate-efficient PAM6 mapper transmits five information bits using two PAM6 symbols:
  - two PAM6 symbols provide
    $6^2 = 36$ possible two-symbol signal states
  - whereas five bits require only
    $2^5 = 32$ states
  - this mapper therefore selects 32 of the 36 available PAM6 pairs and
    leaves four states unused: $36 - 32 = 4$
  - the unused states represent
    $log2(6) - \frac{5}{2} = 0.085$ bits of excess signal-space capacity per PAM6 symbol
- The available excess capacity can become useful when the mapping is extended over multiple PAM6 symbols
  - over 12 symbols: space increase $by [\sqrt{\frac{36}{32}}]^{12} = 2.0 \Rightarrow "bit"$
  - over 20 symbols: space increase $by\ [\sqrt{\frac{36}{32}}]^{20} = 3.2 \Rightarrow "trit"$
  - over 24 symbols: space increase $by\ [\sqrt{\frac{36}{32}}]^{24} = 4.1 \Rightarrow "quad"\ or\ 2\ bits$
  - over 28 symbols: space increase $by\ [\sqrt{\frac{36}{32}}]^{28} = 5.2 \Rightarrow "quint"$

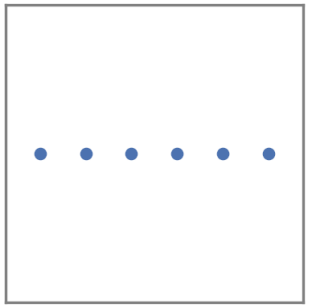
Framed cross 32



Cross 32



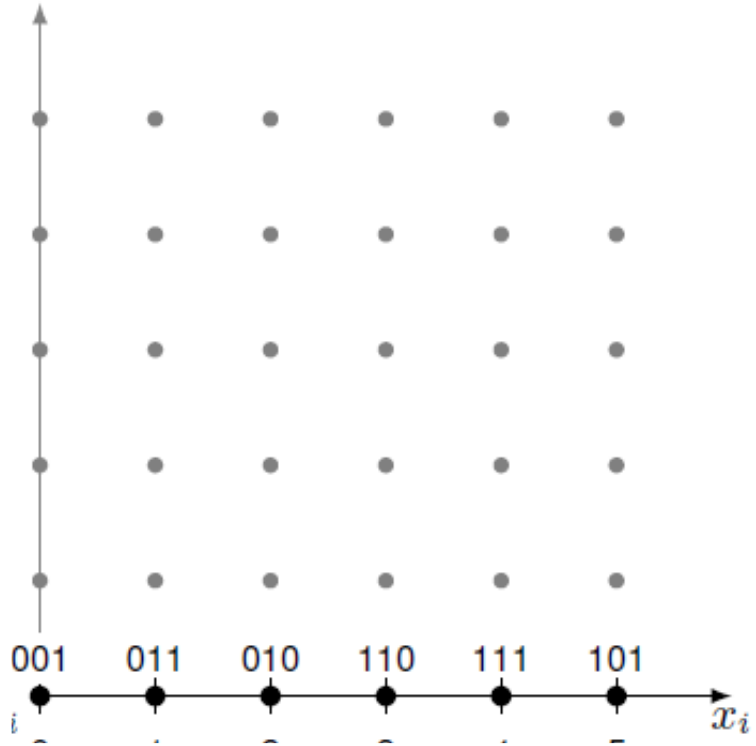
Uniform



# Generalized Constellation Mapping

## Example: PAM6 mapping over 12 symbols

- For a block of 12 PAM6 symbols, the total number of possible signal sequences is
  $$6^{12} = 2.176.782.336$$
- This is slightly greater than the number of combinations required to represent 31 bits
  $$2^{31} = 2{,}147{,}483{,}648$$
- The 12-symbol PAM6 space is large enough to impose one binary constraint while retaining at least $2^{30}$ valid sequences. The resulting code carries 30 payload bits in 12 PAM6 symbols, or 2.5 payload bits per symbol.
- A conventional 5-bit-per-2-symbol mapper would carry only
  $\frac{5}{2} \cdot 12 = 30$ payload bits over these 12 symbols
- The remaining signal-space capacity is therefore sufficient to introduce one bit of structured redundancy without increasing the PAM order, symbol rate, or transmitted peak amplitude
- We propose to exploit this otherwise unused PAM6 signal-space capacity through PAM6 Constraint-Coded Modulation, or PAM6-CCM.
  - In the proposed scheme, 30 independent payload bits are supplemented by one dependent parity bit, forming a rate $= \frac{30}{31}$ coding
  - 30 payload bits + 1 parity bit→12 PAM6 symbols.



# Generalized Constellation Mapping

## Encoder algorithm: mod-2

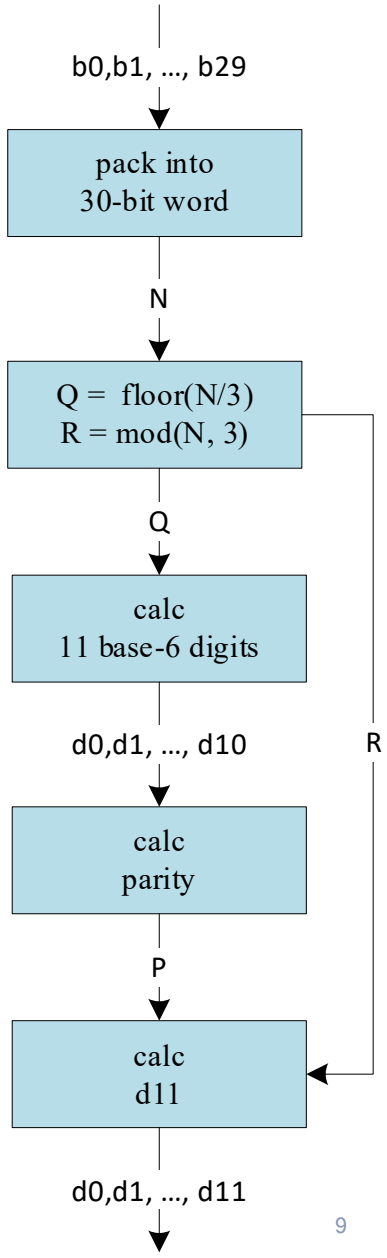
- Let the 30-bit payload represent:
  $$N \in \{0, \ldots, 2^{30} - 1\}$$
- Decompose $N$ as:
  $$Q = \left\lfloor \frac{N}{3} \right\rfloor \text{ and } R = mod(N, 3)$$
- Because
  $$\left\lfloor \frac{2^{30}-1}{3} \right\rfloor = 357{,}913{,}941 < 6^{11} = 362{,}797{,}056$$
- $Q$ always fits into exactly 11 base-6 digits:
  $$Q = (d_0 d_1 d_2 \ldots d_{10})$$
- Calculate their parity:
  $$P = mod(\sum_{n=0}^{10} d_n, 2)$$
- To make the total 12-digit PAM6 index sum even, the final digit must have parity $P$ :
  $$mod(d_{11}, 2) = P$$
- The remainder $R \in \{0,1,2\}$ selects one of the three digits having that parity:
  $$d_{11} = 2 \cdot R + P$$
- Therefore:

| $P$ | $d_{11}$ | |
|---|---|---|
| 0 | {0,2,4} | |
| 1 | {1,3,5} | |

Constraint:

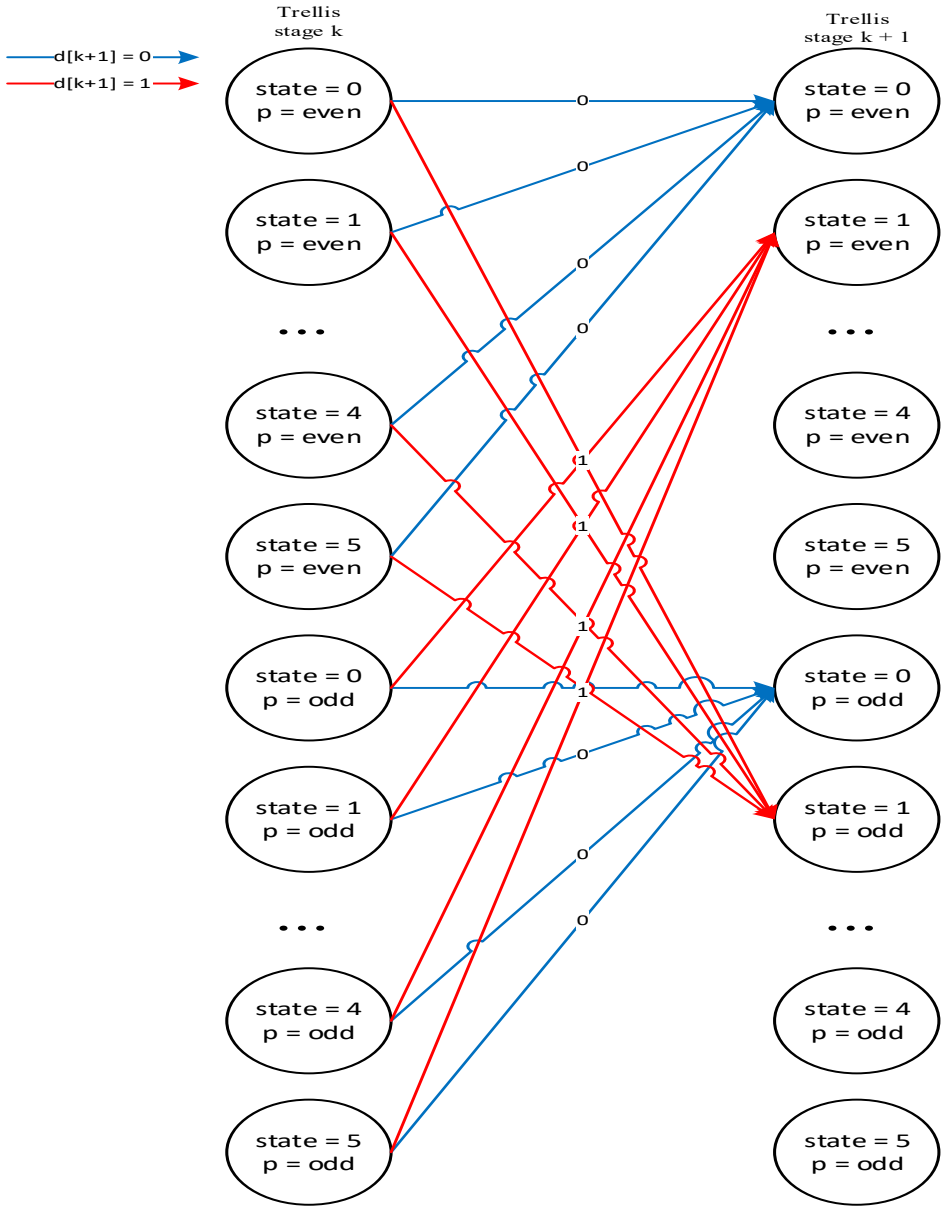
$$mod(\sum_{n=0}^{11} d_n, 2) = 0$$

| $R$ | $P$ | $d_{11}$ |
|---|---|---|
| 0 | 0 | 0 |
| 1 | 0 | 2 |
| 2 | 0 | 4 |
| 0 | 1 | 1 |
| 1 | 1 | 3 |
| 2 | 1 | 5 |



# Joint Channel and Constraint Trellis

## Product-State Trellis for PAM6, Two-Tap PR Channel, and Mod-2 Constraint




### The Trellis diagram:

Presents the ML estimation of the TX PAM sequence based on the received Partial Response (PR) output, 2-tap MLSD $PR(D) = 1 + \alpha \cdot D$ and mod-2 constraint

$$y[k+1] = x(d[k+1]) + \alpha \cdot x(d[k]) + v[k+1]$$

### The update equations

$$s[k+1] = d[k+1]$$

$$p[k+1] = p[k] \oplus d[k+1] = mod(p[k] + d[k+1], 2)$$

### The branch legend:

Blue: $d[k+1] = 0,2,4$ -> parity unchanged

Red: $d[k+1] = 1,3,5$ -> parity toggled

| Current parity | $d = 0,2,4$ | $d = 1,3,5$ |
|---|---|---|
| Even | Even | Odd |
| Odd | Odd | Even |

# Joint Channel and Constraint Trellis

## Performance – AWGN channel

- Uncoded PAM6 minimum distance
  - Without a constraint, two sequences may differ by one adjacent PAM level in one position
    $e = [1]$
  - Its squared distance is
    $d^2_{min,uncoded} = 4 \cdot (1^2) = 4$
- Mod-2 constraint
  - Assume every valid sequence satisfies
    $\sum_k d_k \equiv 0 \ (mod\ 2)$
  - Every legal codeword-difference event satisfies
    $$\sum_k e_k \equiv 0 \ (mod\ 2)$$
  - Single nearest-neighbor event is rejected
  - Allowed errors
    $e = [1, 1]\ or\ [1, -1]$
  - Its squared distance is
    $d^2_{min,coded} = 4 \cdot (1^2 + 1^2) = 8$
  - Asymptotic or $d_{min}$ gain
    $G = \frac{d^2_{min,coded}}{d^2_{min,uncoded}} = 2, G_{dB} = 10log10(G) \approx 3[dB]$

# Simulation assumptions

The below assumptions are same for all the simulations presented below

Modulation PAM6:

$level = [-5{:}\,2{:}\,5]$

Assuming 36/32 mapping, e.g., cross-32 or frame cross-32

$nbits_{sym} = 2.5$

Scan parameter:

PSNR – peak SNR

$$PSNR = 10log10\left(\frac{peak_{pwr}}{\sigma^2}\right) = 10log10\left(\frac{\max(level^2)}{\sigma^2}\right)$$

$$\frac{E_b}{N_0} = 10log10\left(\frac{mean(level^2)}{2\sigma^2 \cdot nbits_{sym}}\right)$$

MLSD:

$PR = 1 + \alpha \cdot D$, examples: $\alpha = 0.2, 0.7$

Code constraint:

Examples: mod-2, 3, 4, 5

*PR channel:*

$$h = \frac{1}{\sqrt{1+\alpha^2}}[1\ \alpha]$$

| Metric | Description | |
|---|---|---|
| Payload BER | Overall bit-error-ratio | |
| BLER | Code-block error ratio | |
| RS SER | Reed-Solomon symbol (10 bits) error ratio | |
| Wrong RS symbol per failed code block | Average number of failed RS symbols per failed code block | |

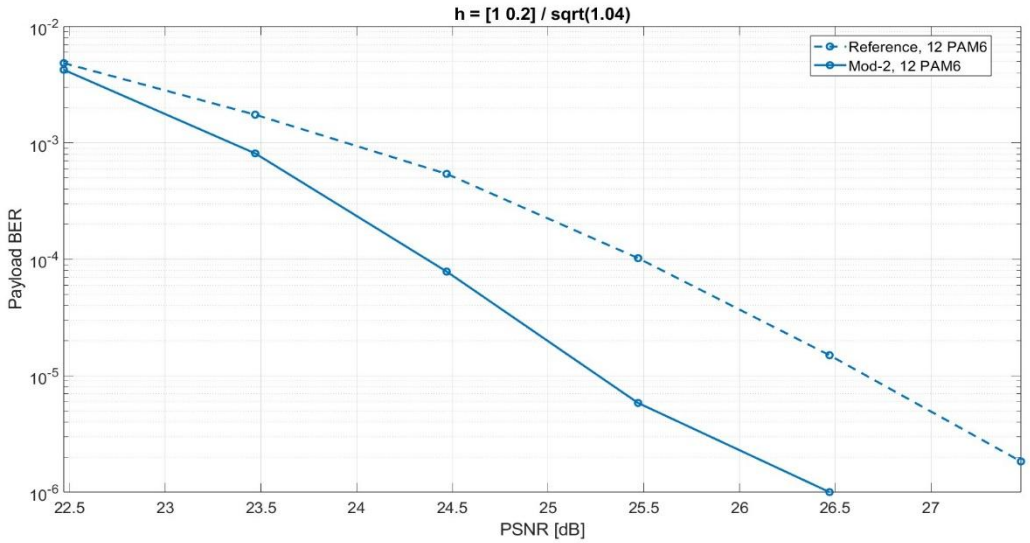
Evaluated metrics

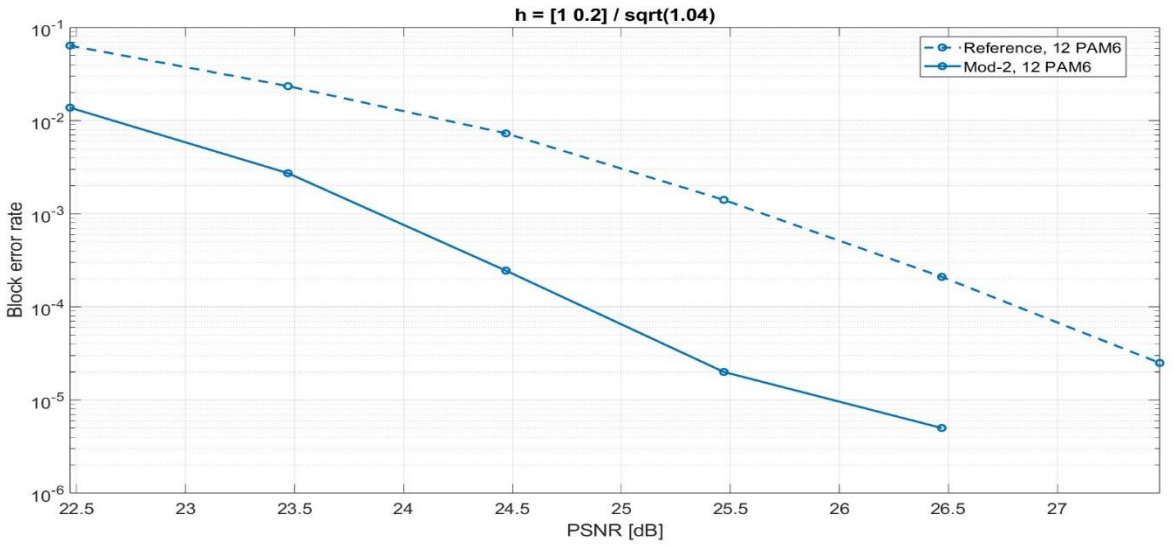
Performance comparison:

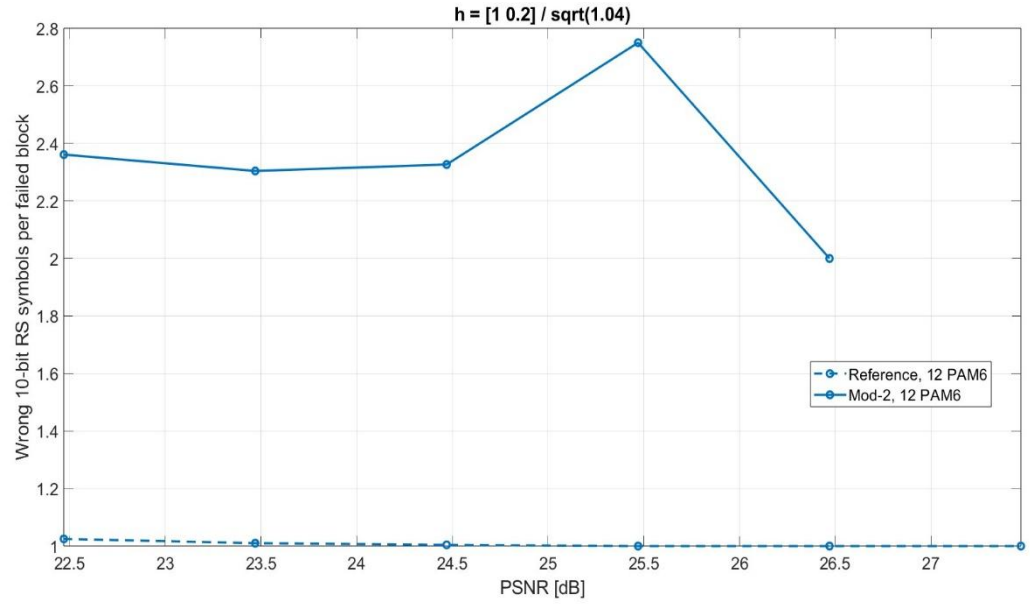
Evaluation is based on RS-SER, assuming ideal RS-codeword interleaving such that RS-symbol errors are independent.

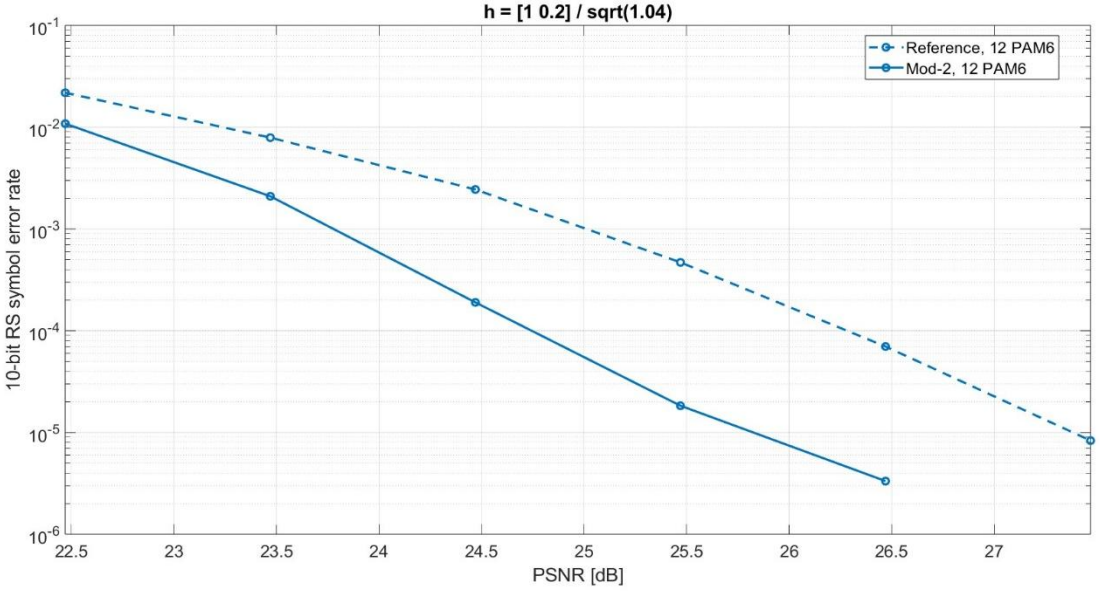
# Native-Excess-Capacity-Only

## Performance simulation: PR $\alpha$ = 0.2, mod-2



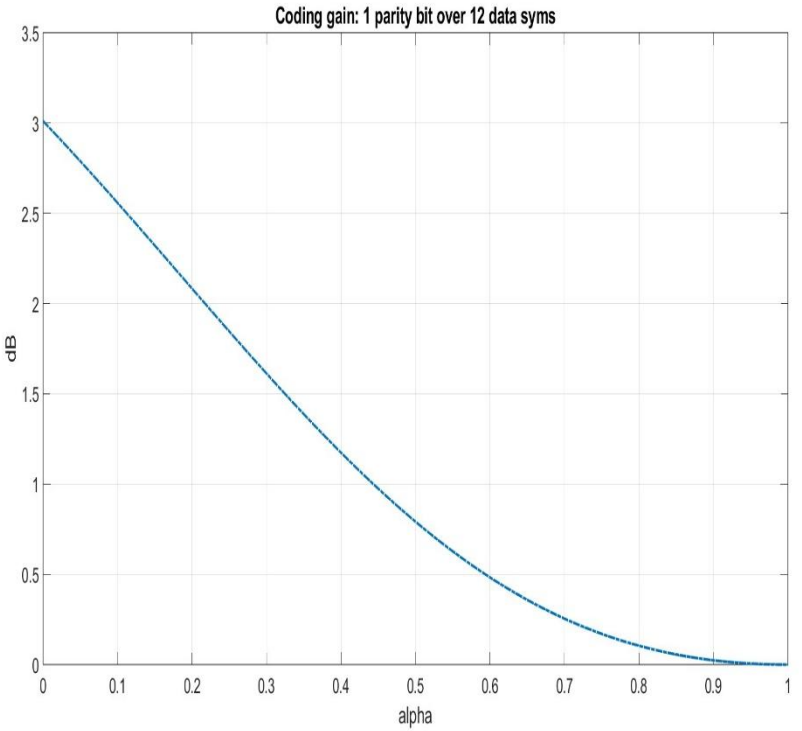



# Joint Channel and Constraint Trellis

Performance: $PR(D) = 1 + \alpha \cdot D$, coding = mod-2

- Uncoded PAM6 minimum distance
  - Assume - same as AWGN
  - Allowed error:
    $e = [1]$
  - Its squared distance is
    $d^2_{min,uncoded} = 4 \cdot (1^2 + \alpha^2)$
- Mod-2 constraint
  - Assume - same as AWGN
  - Allowed errors:
    $e = [1,1]\ or\ [1,-1]$
  - $\bar{d} = conv(\overline{pr}, \bar{e}) =$
    $[1,1]$: $conv([1\ \alpha],[1,1]) = [1\ (1+\alpha)\ \alpha]$
    $[1,-1]$: $conv([1\ \alpha],[1,-1]) = [1\ (1-\alpha)\ -\alpha]$
  - Its squared distance is
    $d^2_{min,coded} = 4 \cdot (1^2 + (1-\alpha)^2 + \alpha^2) = 4 \cdot (1 + \alpha^2 + (1-\alpha)^2)$
  - Asymptotic or $d_{min}$ gain
    $G = \frac{d^2_{min,coded}}{d^2_{min,uncoded}} = \frac{1+\alpha^2+(1-\alpha)^2}{(1^2+\alpha^2)}$
    Example: $G_{dB}(\alpha = 1) = 0[dB]$

| $\alpha$ | Asymptotic $d^2_{\min}$ gain |
|---|---|
| 0 | 3.010 dB |
| 0.5 | 0.792 dB |
| 0.6 | 0.5 dB |
| 0.7 | 0.255 dB |
| 1.0 | 0 dB |



# Higher Moduli Constraint

## Encoder algorithm: mod-S

- Generalization of the 30-bit/12-PAM6-symbol mod- $S$ construction, with $S = 2$.
- Let the n-bit payload represent:

$N \in \{0, \dots, 2^n - 1\}$

- Decompose $N$ as:

$Q = \left\lfloor \frac{N}{L} \right\rfloor$ and $R = mod(N, L)$

Constraint:

$$mod\left(\sum_{i=0}^{m-1} w[i] \cdot d[i], S\right) = 0$$

- $p$ is the number of terminal PAM6 symbols
- $L = \left\lfloor \frac{6^p}{S} \right\rfloor$
- Checksum $C = mod(-\sum_{i=0}^{m-p-1} w[i] \cdot d[i], S)$
- PAM6 symbol indices:

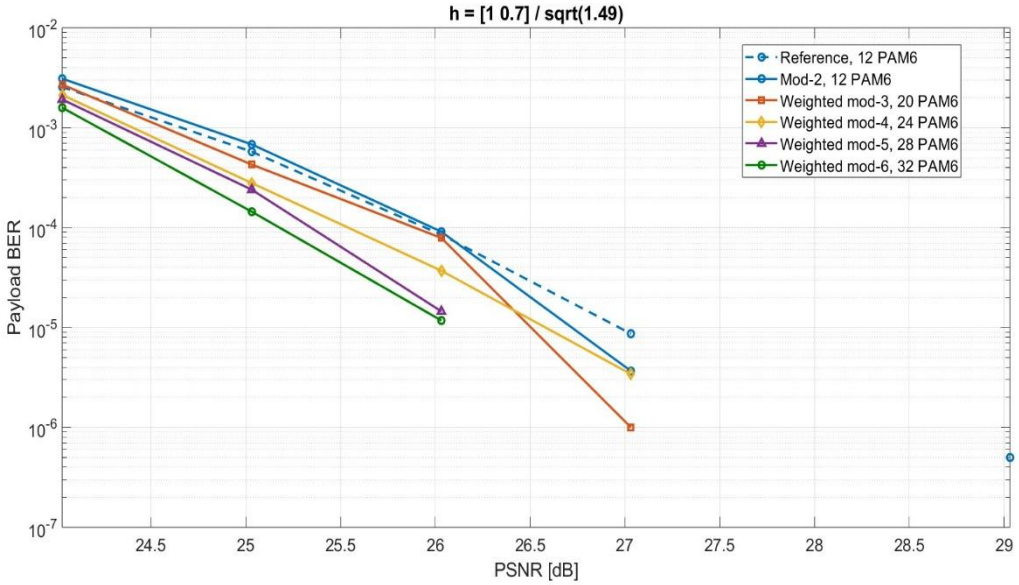
$d[0: m-p-1] = base6_{m-p}(Q), J = R \cdot S + C$

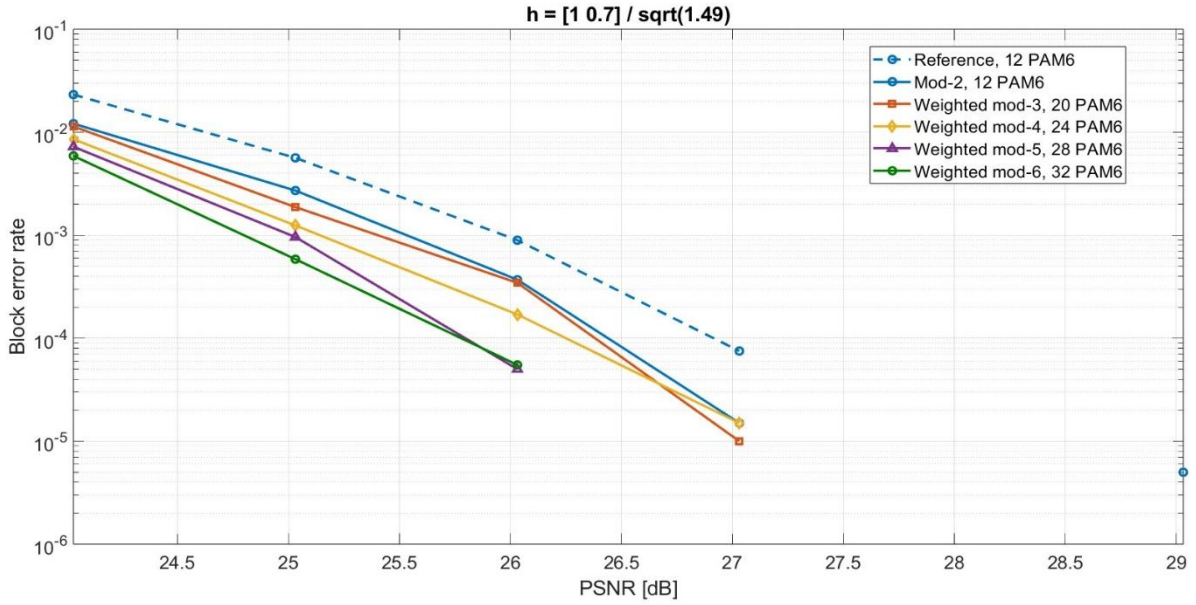
$d[m-p: m-1] = base6_p(J)$

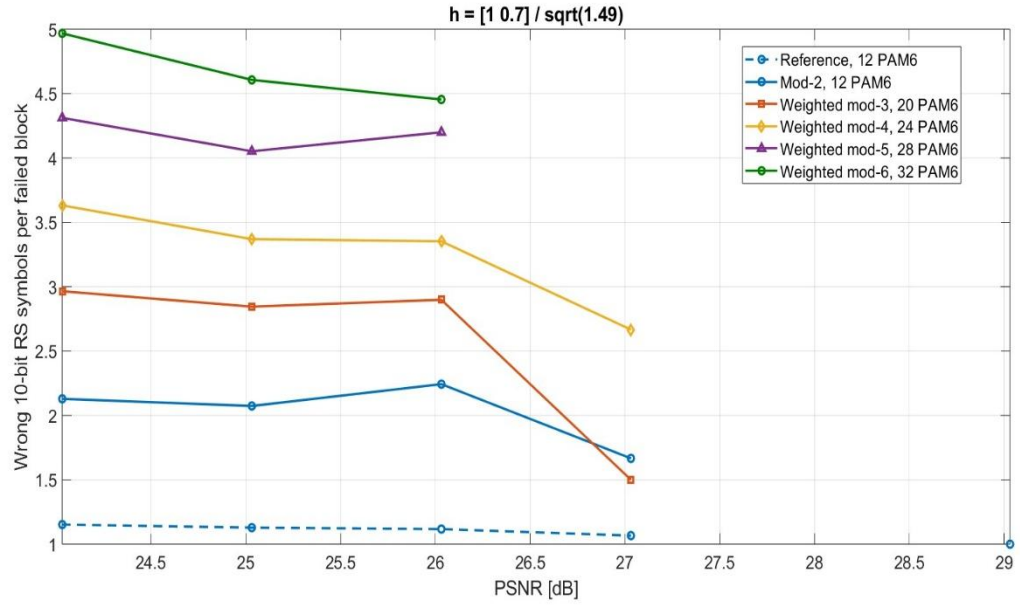
| Modulo encoding $S$ | n-data bits | m-PAM6 symbols | k-RS symbols | $L = \left\lfloor \frac{6^p}{S} \right\rfloor$ | Decompose integer Q | Decompose $R \in [0: L-1]$ | $C \in [0: S-1]$ | Weights example |
|---|---|---|---|---|---|---|---|---|
| 2 | 30 | 12 | 3 | $\left\lfloor \frac{6^1}{2} \right\rfloor = 3$ | 11 PAM6 symbols | [0: 2] | [0: 1] | [1] x 12 repeat |
| 3 | 50 | 20 | 5 | $\left\lfloor \frac{6^1}{3} \right\rfloor = 2$ | 19 PAM6 symbols | [0: 1] | [0: 2] | [1 2] x 10 repeat |
| 4 | 60 | 24 | 6 | $\left\lfloor \frac{6^2}{4} \right\rfloor = 9$ | 22 PAM6 symbols | [0: 8] | [0: 3] | [1 3] x 12 repeat |
| 5 | 70 | 28 | 7 | $\left\lfloor \frac{6^2}{5} \right\rfloor = 7$ | 26 PAM6 symbols | [0:6] | [0:4] | [1 4] x 14 repeat |
| 6 | 80 | 32 | 8 | $\left\lfloor \frac{6^1}{6} \right\rfloor = 1$ | 31 PAM6 symbols | 0 | [0:5] | [1 5] x 16 repeat |

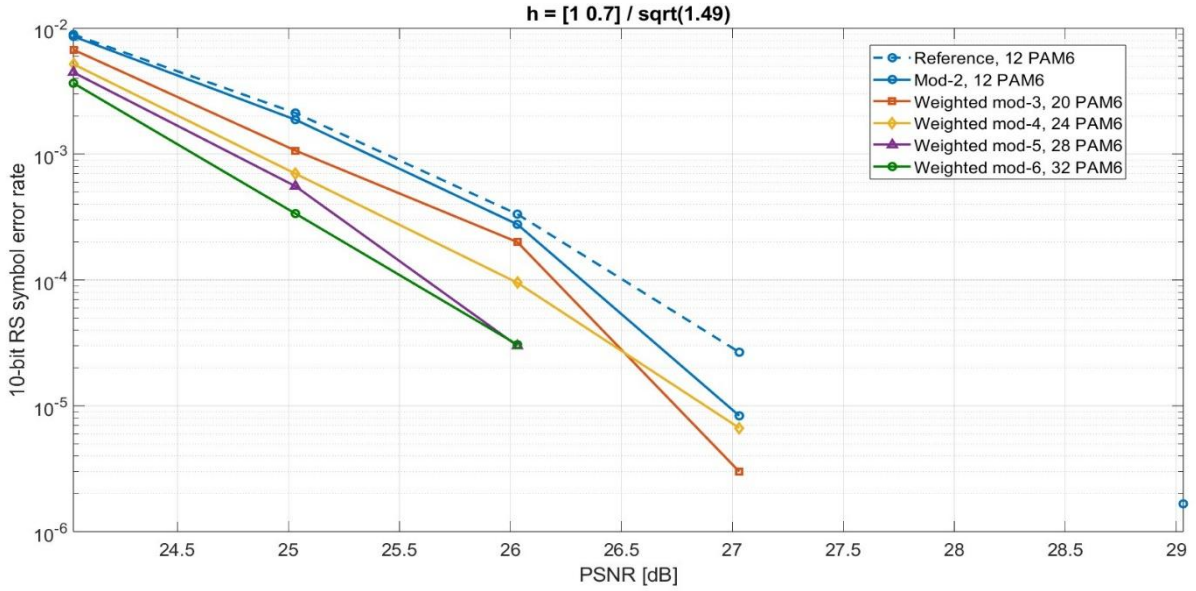
# Higher moduli encoding

## Performance simulation: PR $\alpha$ = 0.7, mod-2:6



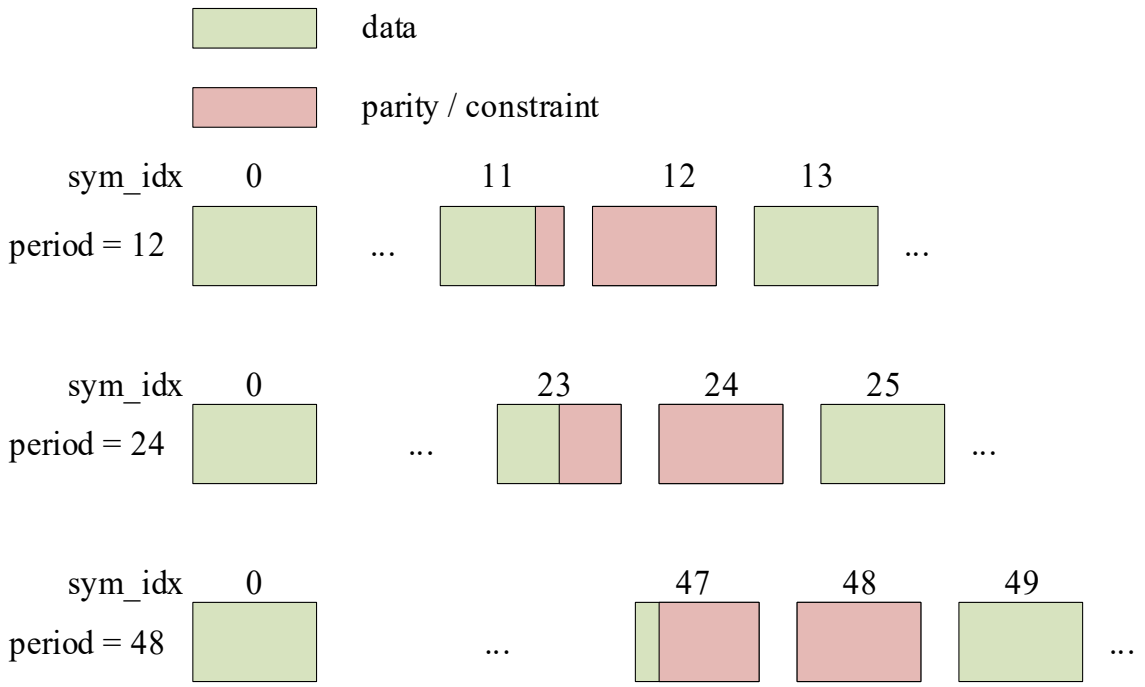



# Hybrid Native-Excess and Supplemental-Resource Coding

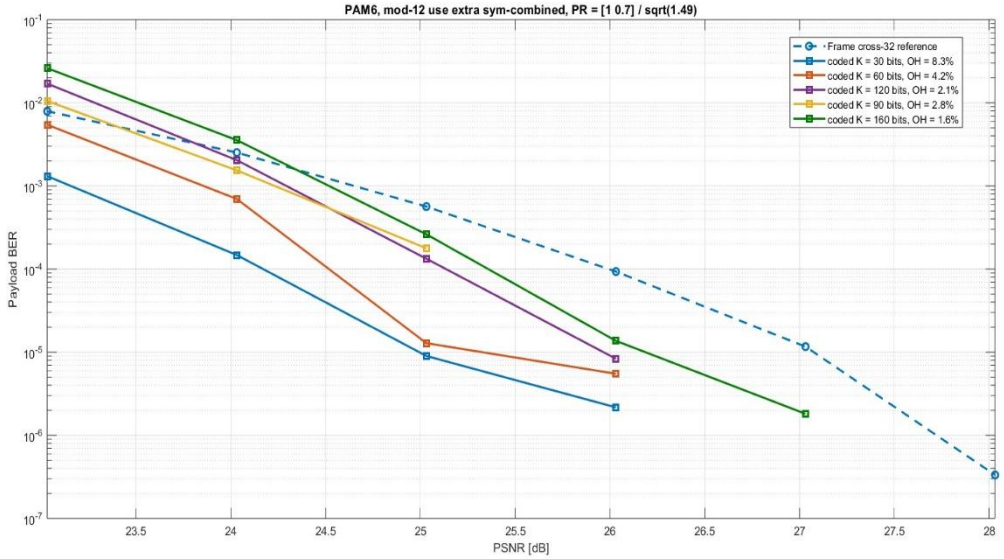
The proposal uses native excess constellation capacity together with supplemental symbols that carry additional constraint information.

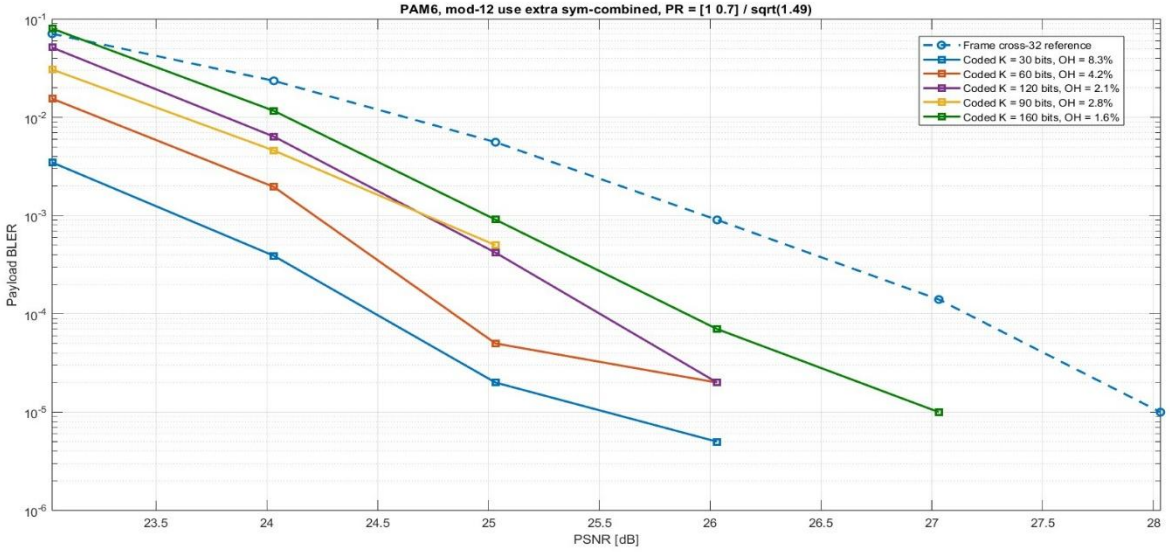
| Block length [bits] | Block length [sym] | OH [%] | Space increase factor | Integer number of bits |
|---|---|---|---|---|
| 30 | 12 + 1 = 13 | 8.3 | 12 | 3 |
| 60 | 24 + 1 = 25 | 4.2 | 24 | 4 |
| 80 | 32 + 1 = 33 | 3.1 | 36 | 5 |
| 90 | 36 + 1 = 37 | 2.8 | 48 | 5 |
| 120 | 48 + 1 = 49 | 2.1 | 96 | 6 |
| 160 | 64 + 1 = 65 | 1.6 | 256 | 8 |

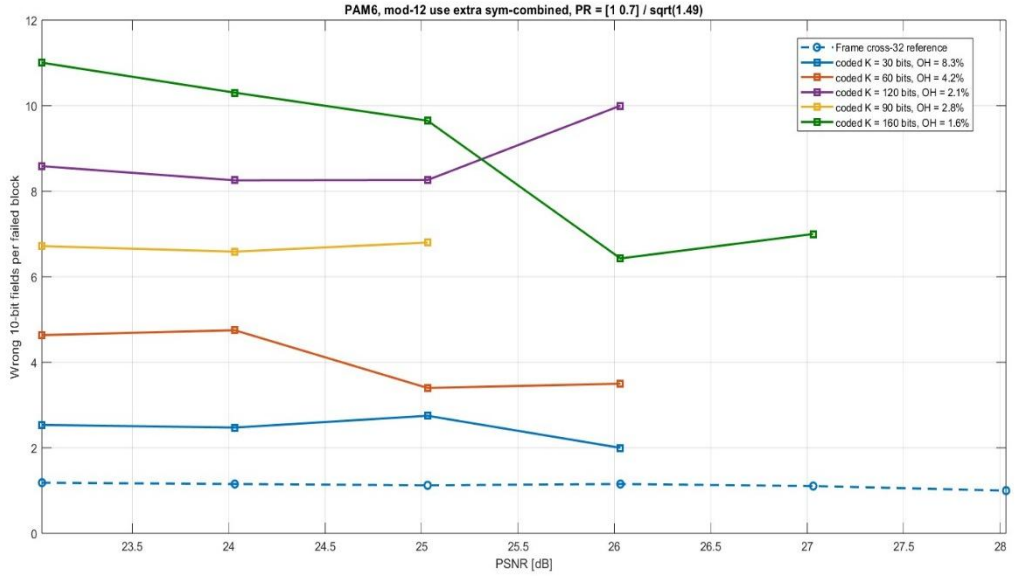


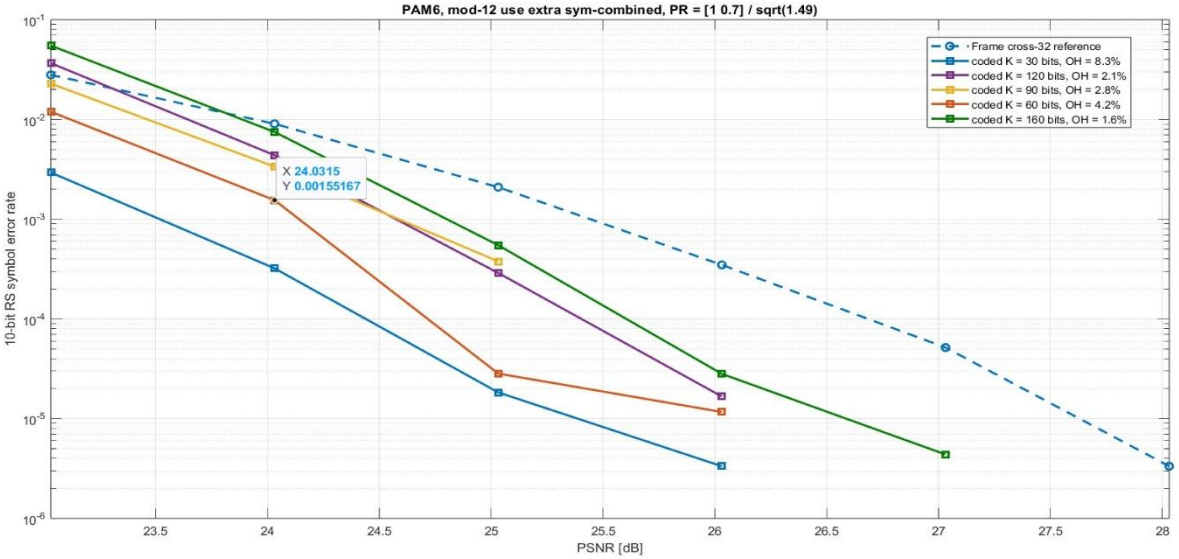
# Hybrid Native-Excess and Supplemental-Resource Coding

## Performance simulation: PR $\alpha$ = 0.7, mod-12



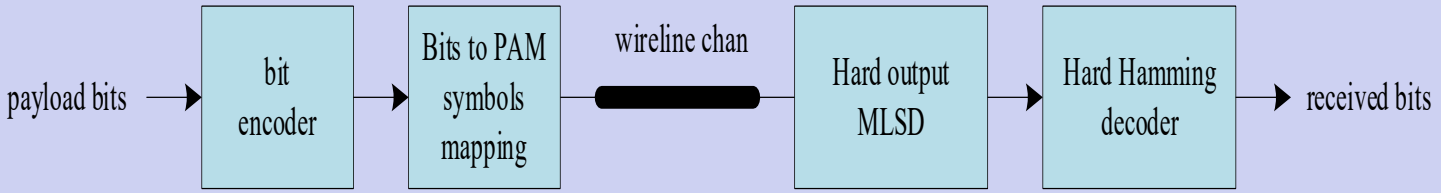
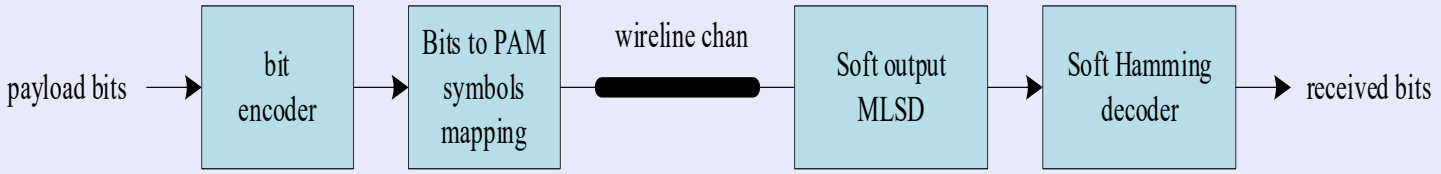
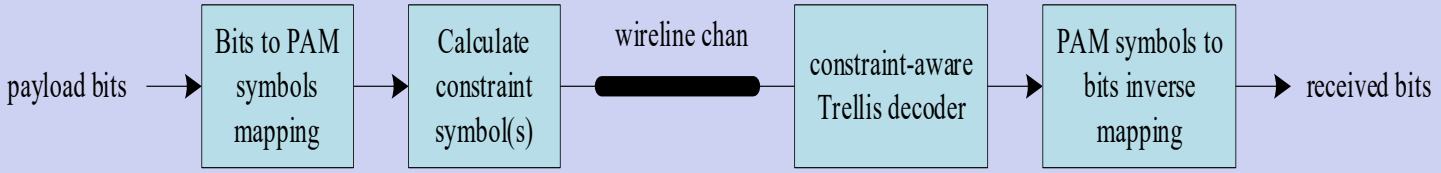



# Comparison with the Conventional Coding scheme

The major difference between the conventional and the new proposed scheme

- Conventional: separate demodulation and decoding (SDD)
- New proposal: joint demodulation and decoding (JDD)

| Coding option | Encoder and decoder | Demodulator and decoder |
|---|---|---|
| Hard Hamming |  | • hard MLSD<br>• hard Hamming decoder |
| Soft Hamming |  | • soft MLSD (SOVA, BCJR, List-based sequence detection)<br>• soft Hamming decoder |
| Constraint-Coded Modulation |  | • create Product-state trellis and<br>• perform MLSD demodulation |

# Simulation assumptions

## Modulation PAM6:

$level = [-5:2:5]$

## Assuming 36/32 mapping, e.g., cross-32 or frame cross-32

$nbits_{sym} = 2.5$

## Scan parameter:

PSNR – peak SNR

$$PSNR = 10log10\left(\frac{peak_{pwr}}{\sigma^2}\right) = 10log10\left(\frac{\max(level^2)}{\sigma^2}\right)$$

$$\frac{E_b}{N_0} = 10log10\left(\frac{mean(level^2)}{2\sigma^2 \cdot nbits_{sym}}\right)$$

## MLSD:

$PR = 1 + \alpha \cdot D, \alpha = 0.7$

| Metric | Description |
|---|---|
| Payload BER | Overall bit-error-ratio |
| BLER | Code-block error ratio |
| RS SER | Reed-Solomon symbol (10 bits) error ratio |
| Wrong RS symbol per failed code block | Average number of failed RS symbols per failed code block |

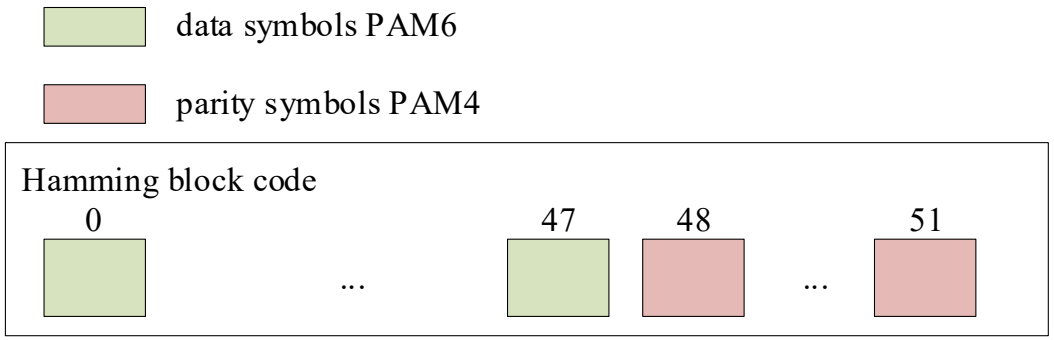
Evaluated metrics

## Encoding:

Extended Hamming block (128, 120)

- 48 PAM6 data symbols = 120 bits
- 4 PAM4 parity symbols = 8 bits

Interleaving: disable




## Decoding:

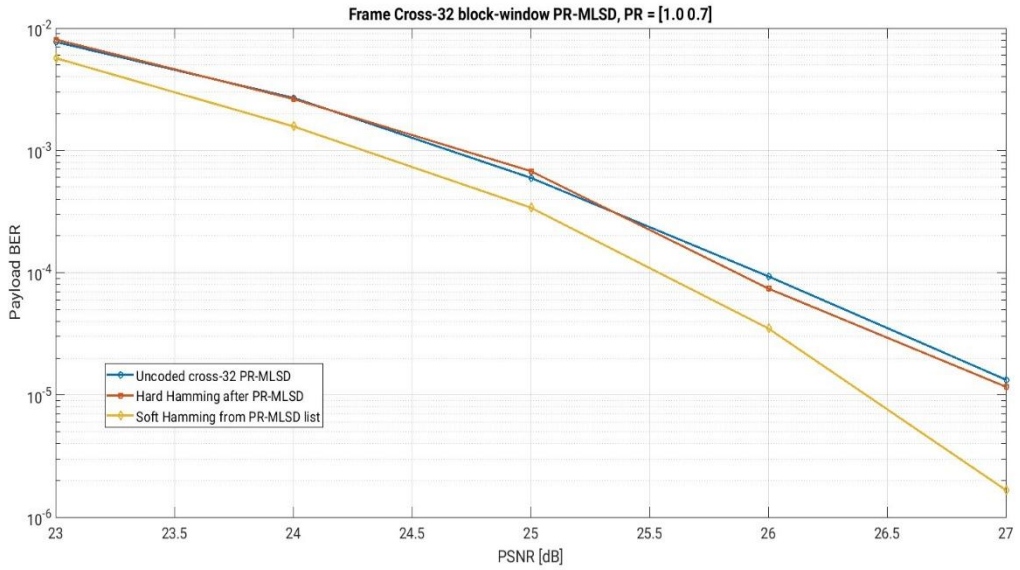
- K-best list size = 16
- number of Chase bits = 8

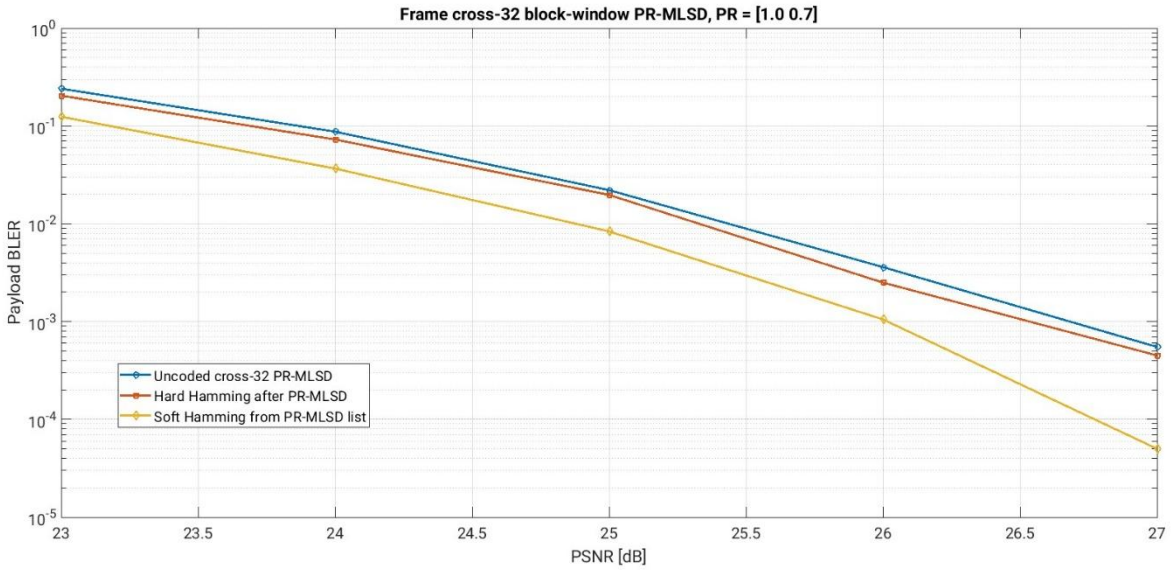
## Signaling overhead:

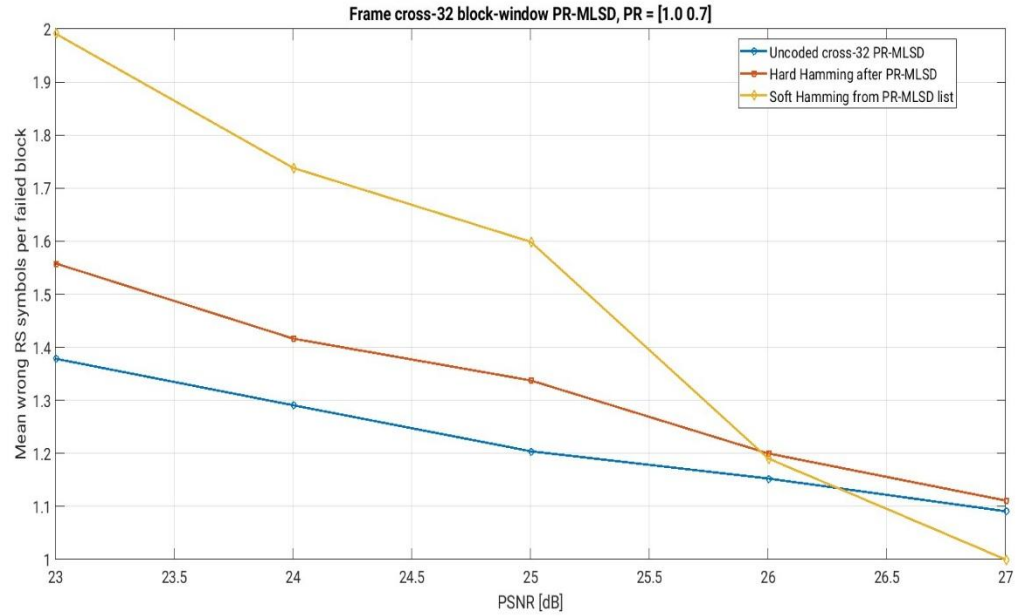
The Hamming frame uses 52 symbol periods versus 48 for uncoded PAM6, corresponding to 8.33% supplemental-symbol overhead. The plotted comparison uses equal peak amplitude and does not include the additional bandwidth penalty required at equal payload throughput.

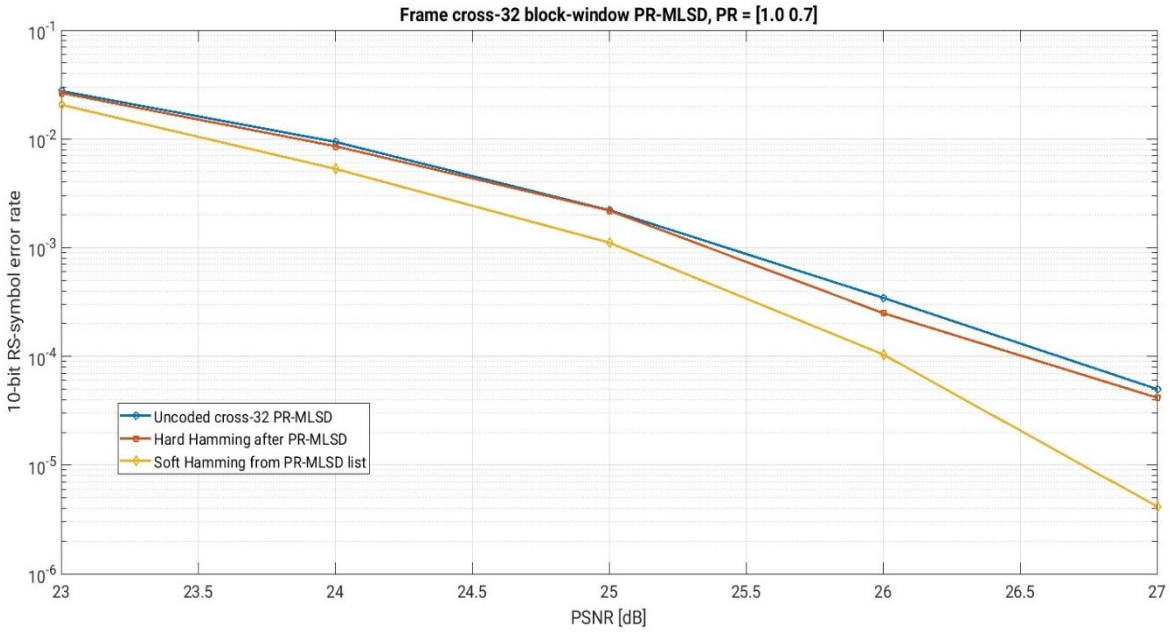
# Hamming code: Hard vs Soft demodulation and decoding

## Performance simulation: PR $\alpha$ = 0.7



# Summary – performance analysis

## Native-capacity-only CCM: 0% supplemental resource overhead

| Test case | Performance gain [dB] | Wrong RS symbols per failed block |
|---|---|---|
| uncoded | 0 | 1.2 |
| Mod-2 | 0.2 | 2.3 |
| Mod-3 | 0.3 | 2.9 |
| Mod-4 | 0.5 | 3.4 |
| Mod-5 | 0.9 | 4.2 |
| Mod-6 | 1.0 | 4.5 |

Native-capacity mod-6 provides up to 1.0 dB gain without additional transmitted symbols or baud-rate overhead, at the cost of increased MLSD state processing.

## Hybrid CCM: native capacity plus supplemental signaling resources

| Test case | Performance gain [dB] | Wrong RS symbols per failed block | Supplemental symbol overhead (OH) [%] |
|---|---|---|---|
| uncoded | 0 | 1.2 | 0 |
| K = 30 bits | 2.2 | 2.9 | 8.3 |
| K = 60 bits | 1.9 | 3.4 | 4.2 |
| K = 90 bits | 1.3 | 6.8 | 2.8 |
| K = 120 bits | 1.2 | 8.3 | 2.1 |
| K = 160 bits | 1.1 | 9.7 | 1.6 |

Hybrid configurations provide only 0.1–1.2 dB additional gain beyond mod-6, while requiring 1.6–8.3% resource overhead and producing greater residual-error concentration for longer blocks.

Note: Gain measured at RS-SER = $10^{-4}$, relative to uncoded PAM6, for $PR(D) = 1 + 0.7D$, using equal peak amplitude.

# Conclusion

- We have demonstrated constraint-based encoding for PAM6 in both native-capacity-only and hybrid supplemental-resource configurations.
- Native-capacity mod-6 provides approximately 1.0 dB gain without additional transmitted symbols or baud-rate overhead, at the cost of increased MLSD state processing and survivor memory.
- Hybrid configurations provide additional gain, but the incremental improvement beyond mod-6 is limited when supplemental-resource overhead and residual-error concentration are considered.
- Among the evaluated configurations, native-capacity mod-6 currently appears to provide the most favorable system-level tradeoff.

# Backup

# Conventional Inner-Code Overhead May Not Fit

## Practical channel bandwidth

Example: Hamming (128,120)

$$R_H = \frac{120}{128} = 0.9375$$

Required signaling-rate factor at equal payload throughput:

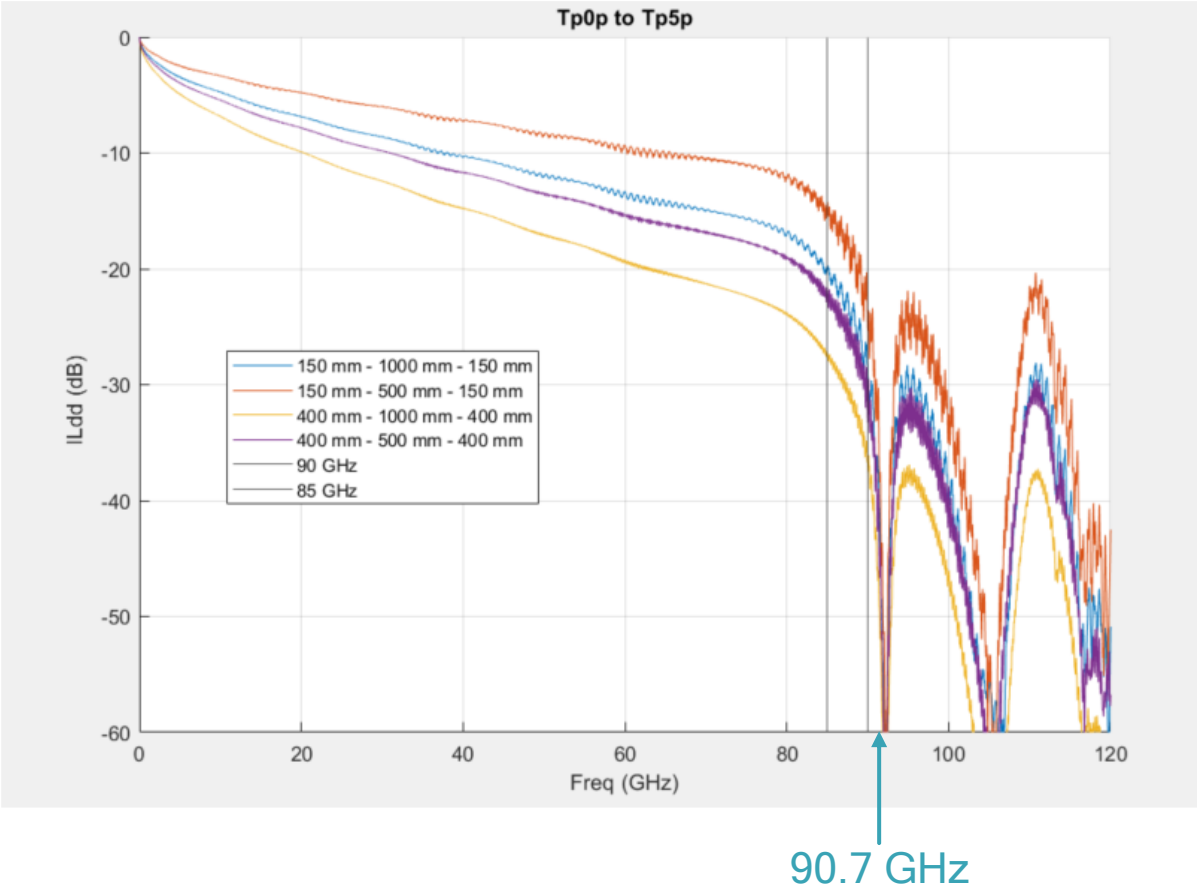
$$\frac{1}{R_H} = \frac{128}{120} = 1.0667$$

Required signaling-rate increase:

$$\frac{Delta_R}{R} = \frac{128}{120} - 1 = 6.67\%$$

Required Nyquist bandwidth:

$$B_H = 85 \cdot \frac{128}{120} \approx 90.7 \text{ GHz}$$




Bandwidth comparison

- Uncoded PAM6: 85 GHz
- Provisional practical maximum: approximately 88 GHz
- Hamming (128,120): approximately 90.7 GHz
- Potential channel notch: approximately 91 GHz

**At equal payload throughput, Hamming (128,120) requires more bandwidth than the assumed channel margin can provide.**

# Higher moduli encoding

## Performance $PR(D) = 1 + \alpha \cdot D$, coding = mod-2, mod-3, mod-4

| | **separated single-step errors** | **alternating and not rejected event ($d_{min}$)** |
|---|---|---|
| Mod-2<br>weight [1] | Two separated single-step errors<br>$[\Delta, 0, \dots, \Delta] \rightarrow [1, \alpha] \rightarrow [1, \alpha, 0, \dots, 0, 1, \alpha] \rightarrow$<br>$d_{min}^2 = 2 \cdot \Delta^2 \ (1 + \alpha^2)$ | Two adjacent single-step errors = $[\Delta, -\Delta]$<br>$[\Delta, -\Delta] \rightarrow [1, \alpha] \rightarrow [1, \alpha - 1, -\alpha] \rightarrow$<br>$d_{min}^2 = \Delta^2(1 + \alpha^2 + (1 - \alpha)^2) = 2 \cdot \Delta^2 \ (1 - \alpha + \alpha^2)$ |
| Mod-3<br>weight [1,2] | Two separated single-step errors<br>$[\Delta, 0, \dots, \Delta] \rightarrow [1, \alpha] \rightarrow [1, \alpha, 0, \dots, 0, 1, \alpha] \rightarrow$<br>$d_{min}^2 = 2 \cdot \Delta^2 \ (1 + \alpha^2)$ | Length-three alternating event = $[\Delta, -\Delta, \Delta]$<br>$[\Delta, -\Delta, \Delta] \rightarrow [1, \alpha] \rightarrow [1, \alpha - 1, 1 - \alpha, \alpha] \rightarrow$<br>$d_{min}^2 = \Delta^2(1 + \alpha^2 + 2(1 - \alpha)^2)$ |
| Mod-4<br>weight [1,3] | Two separated single-step errors<br>$[\Delta, 0, \dots, \Delta] \rightarrow [1, \alpha] \rightarrow [1, \alpha, 0, \dots, 0, 1, \alpha] \rightarrow$<br>$d_{min}^2 = 2 \cdot \Delta^2 \ (1 + \alpha^2)$ | Length-four alternating event = $[\Delta, -\Delta, \Delta, -\Delta]$<br>$[\Delta, -\Delta, \Delta, -\Delta] \rightarrow [1, \alpha] \rightarrow [1, \alpha - 1, 1 - \alpha, \alpha - 1, -\alpha] \rightarrow$<br>$d_{min}^2 = \Delta^2(1 + \alpha^2 + 3(1 - \alpha)^2)$ |

Candidate gain for the illustrated event families:

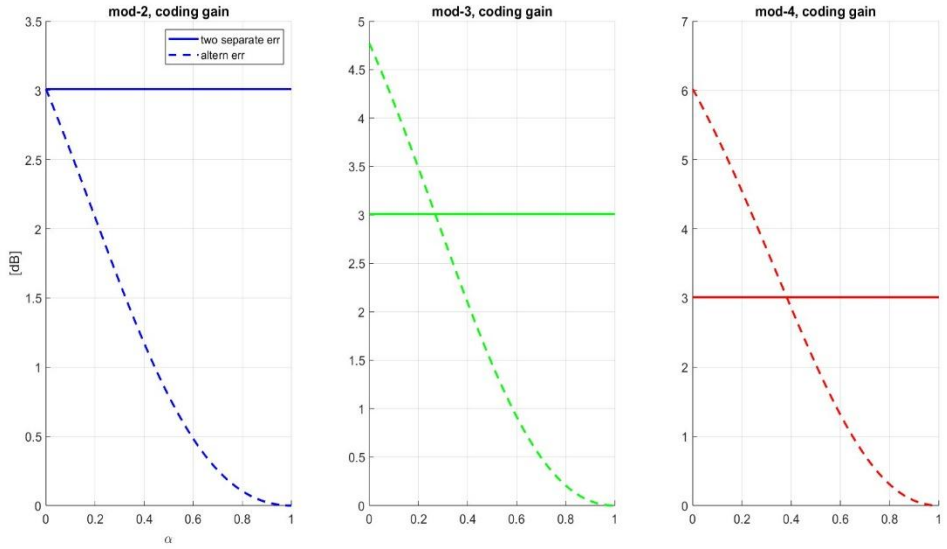
$$Coding\ gain = 10\log10\left[\frac{\min\left(d_{sep}^2, d_{alt}^2\right)}{d_{uncoded}^2}\right]$$

where

$$d_{uncoded}^2 = \Delta^2(1 + \alpha^2)$$

General formula:

$$Coding\ gain(mod) = 10log10\left(1 + F(mod) \cdot \frac{(1 - \alpha)^2}{1 + \alpha^2}\right)$$



# Higher Moduli encoding

## Decoder / Trellis complexity

| Reused | Increased |
|---|---|
| • Channel-output prediction<br>• PAM6 candidate generation<br>• Euclidean-distance evaluation<br>• Channel-state transitions | • State-metric additions<br>• Compare-select operations<br>• Survivor-memory storage<br>• Modulo-state updates<br>• Block-termination checks |

| Modulo encoding | Number of states | unique Euclidean metrics | path-metric additions/extensions | |
|---|---|---|---|---|
| 2 | 6 x 2 = 12 | 6 x 6 = 36 | 12 x 6 = 72 | |
| 3 | 6 x 3 = 18 | 6 x 6 = 36 | 18 x 6 = 108 | |
| 4 | 6 x 4 = 24 | 6 x 6 = 36 | 24 x 6 = 144 | |
| 5 | 6 x 5 = 30 | 6 x 6 = 36 | 30 x 6 = 180 | |
| 6 | 6 x 6 = 36 | 6 x 6 = 36 | 36 x 6 = 216 | |
| | | | | |